SAFIR®
A software for modeling the behavior of structures subjected to the fire

Course by

Jean Marc Franssen & Thomas Gernay

LIÈGE université

JOHNS HOPKINS
WHITING SCHOOL of ENGINEERING

Last revision: 28/01/2020

Basic theory of beam F.E.

1) Local system of coordinates
2) Degrees of freedom
3) The basic equations
4) Material properties
5) Numerical integration on the length
6) Numerical integration on the surface
7) Stiffness in torsion
8) Examples

Three steps in the structural fire design:

1. *Define the fire (not made by SAFIR).*

2. *Calculate the temperatures in the structure.*

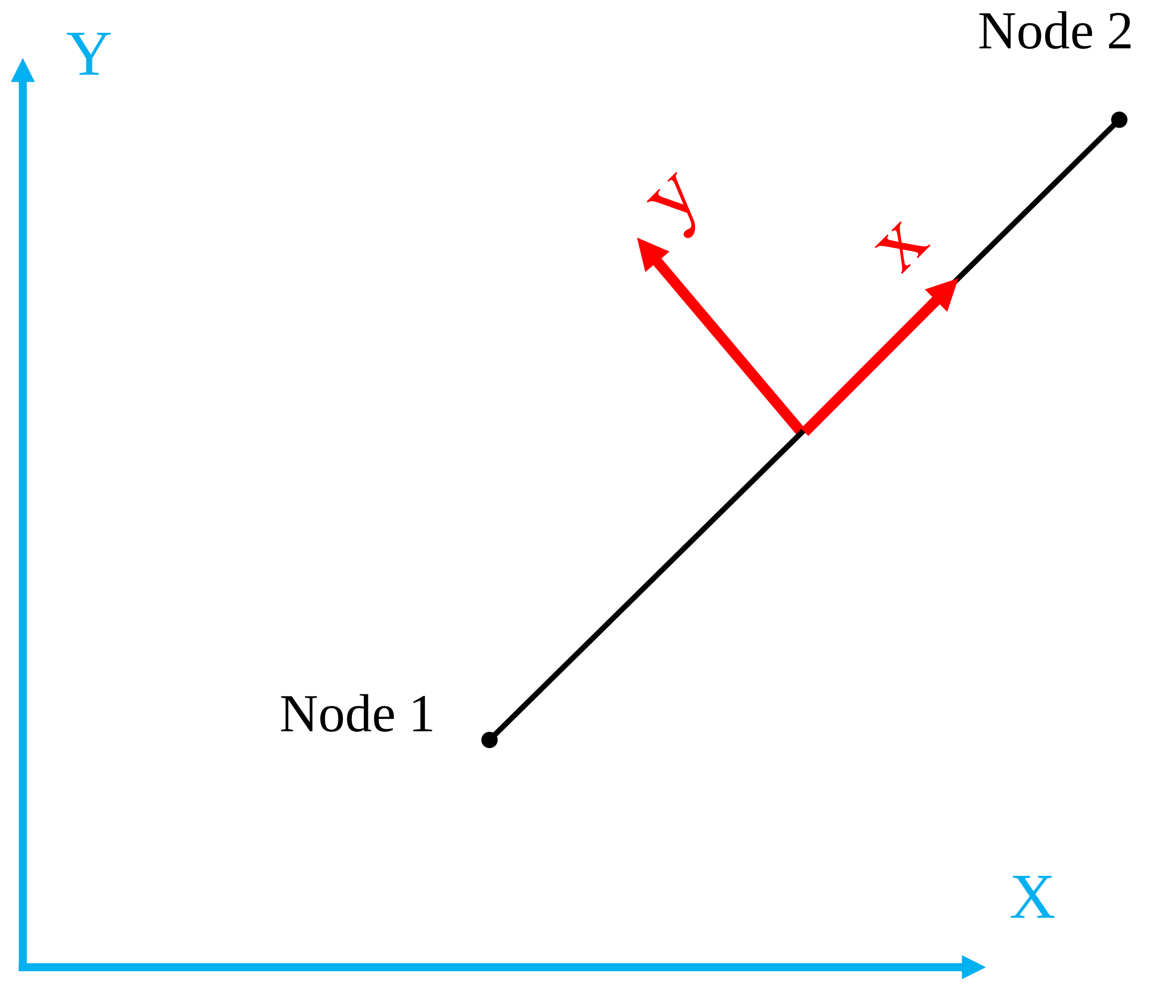
3. *Calculate the mechanical behaviour.*

1) Local system of coordinates
2) Degrees of freedom
3) The basic equations
4) Material properties
5) Numerical integration on the length
6) Numerical integration on the surface
7) Stiffness in torsion
8) Examples

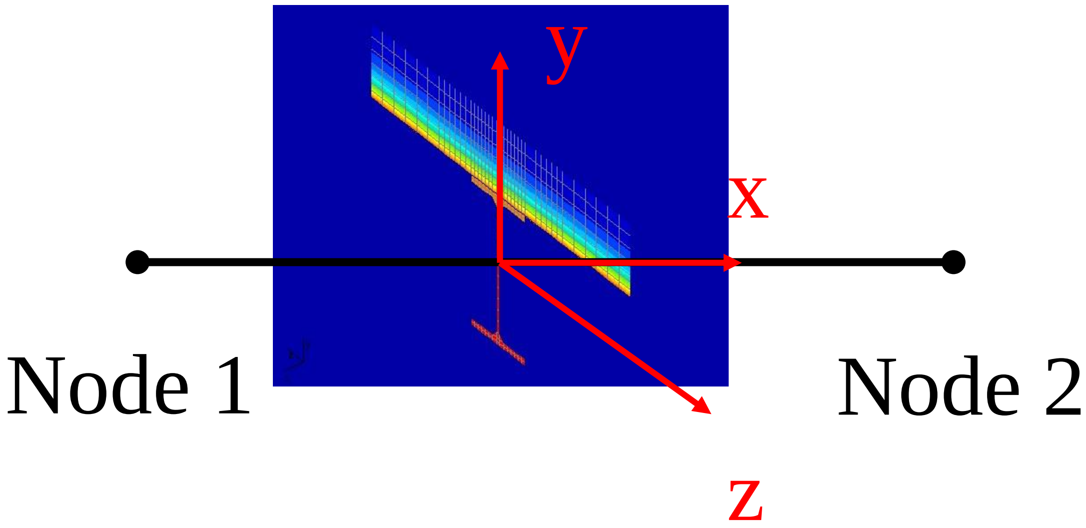
2D beam finite element

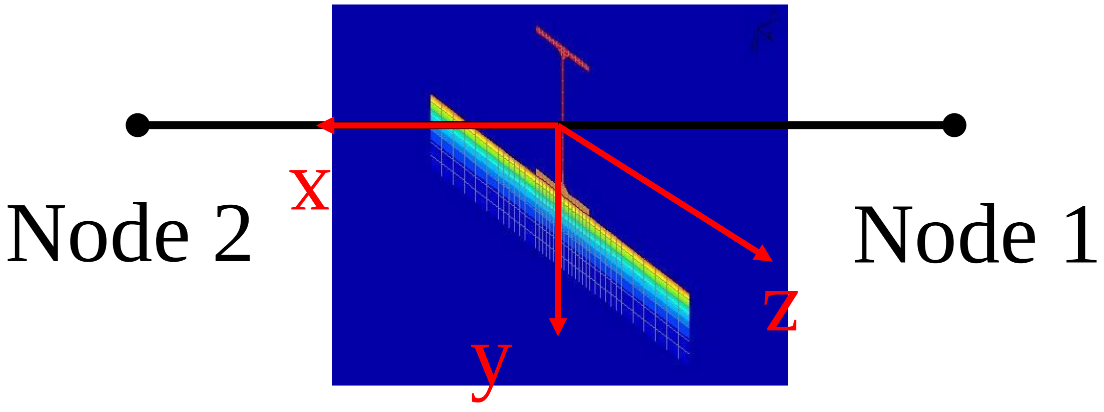
Global system of coordinates (of the structure)

Local system of coordinates (of each beam)

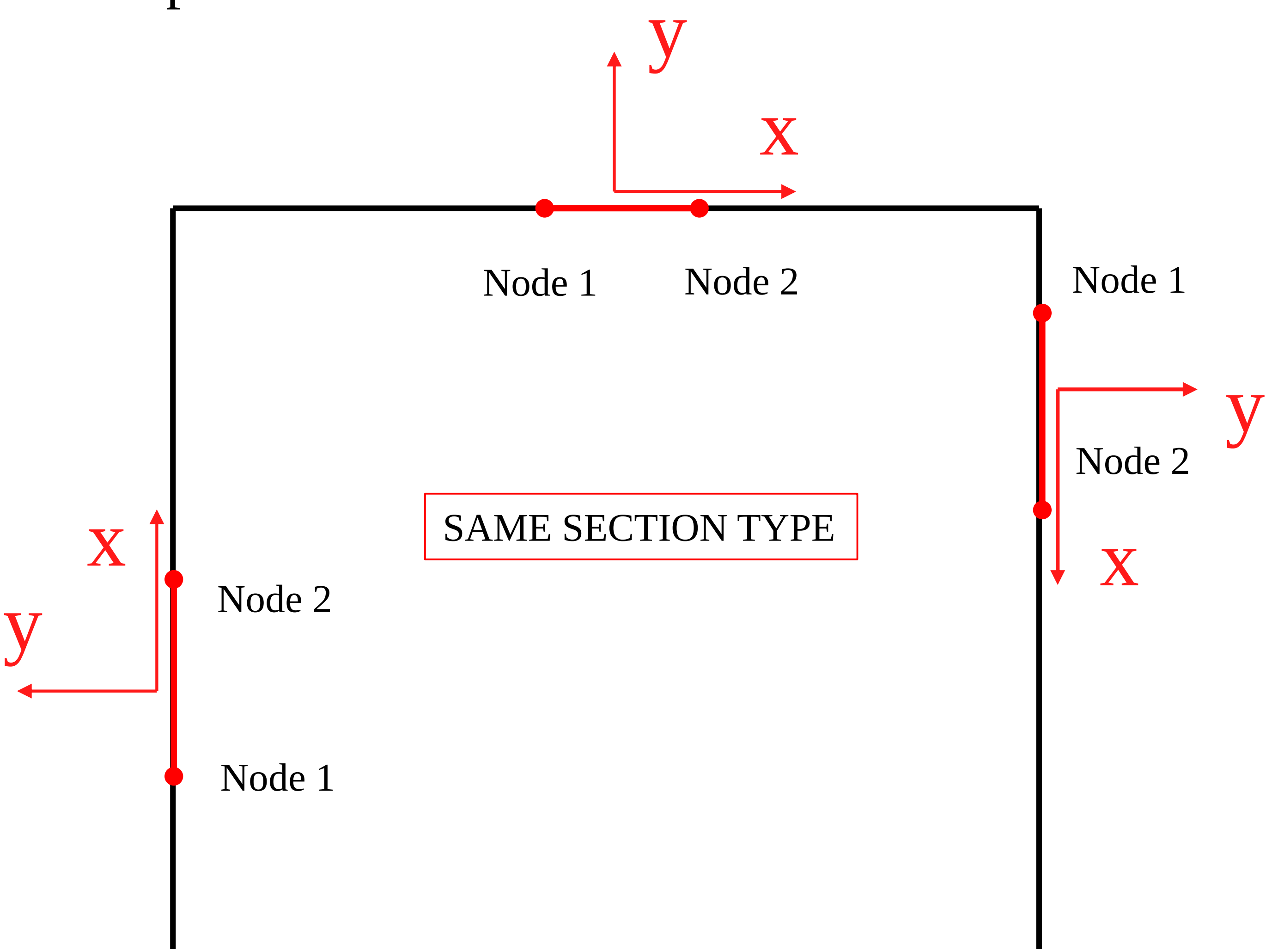


Why is the local SoC important?





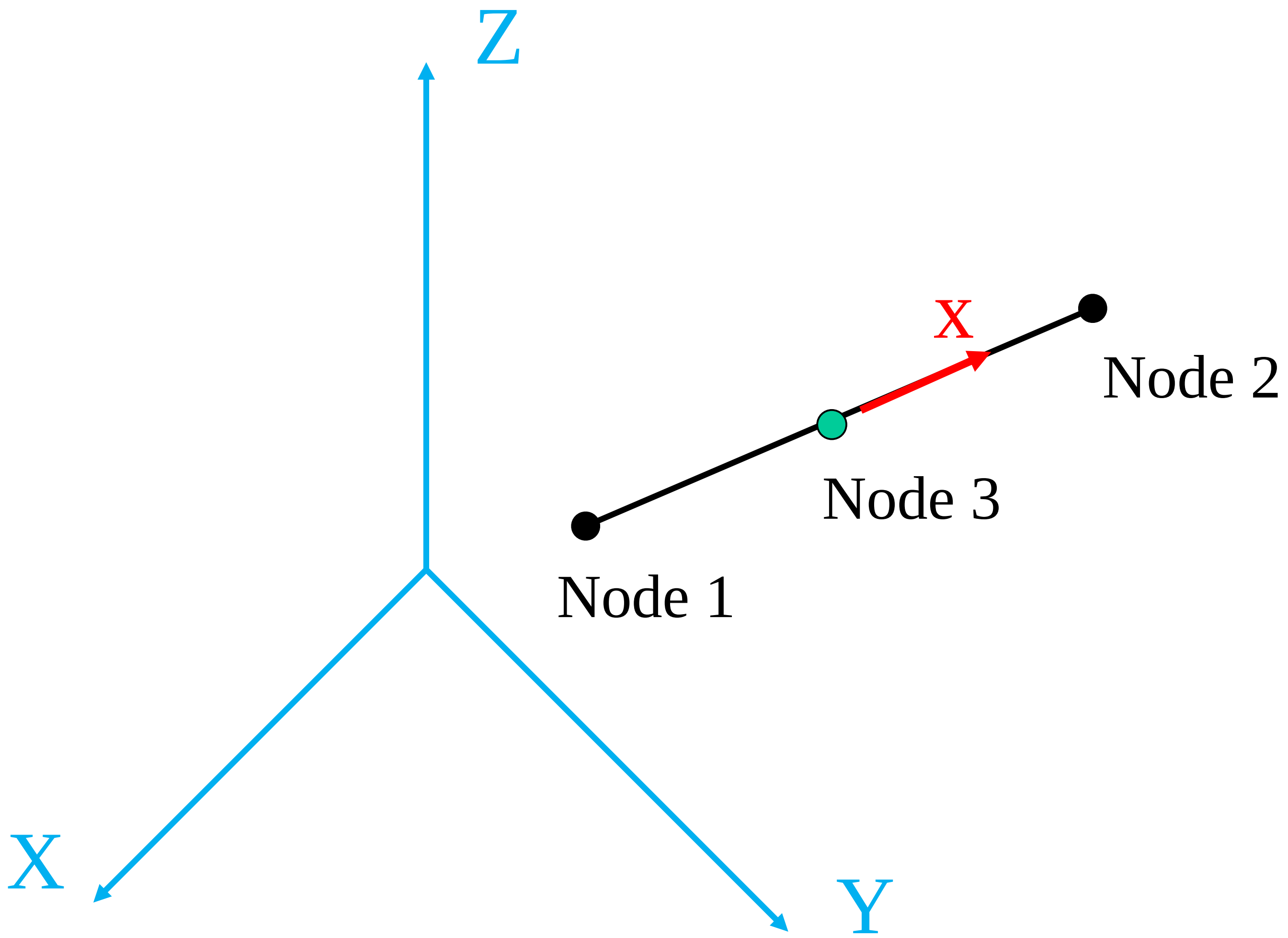
Example: a simple 2D frame.



The same section type can be used because the unexposed side of the section (positive values of y) is correctly located toward the outside.

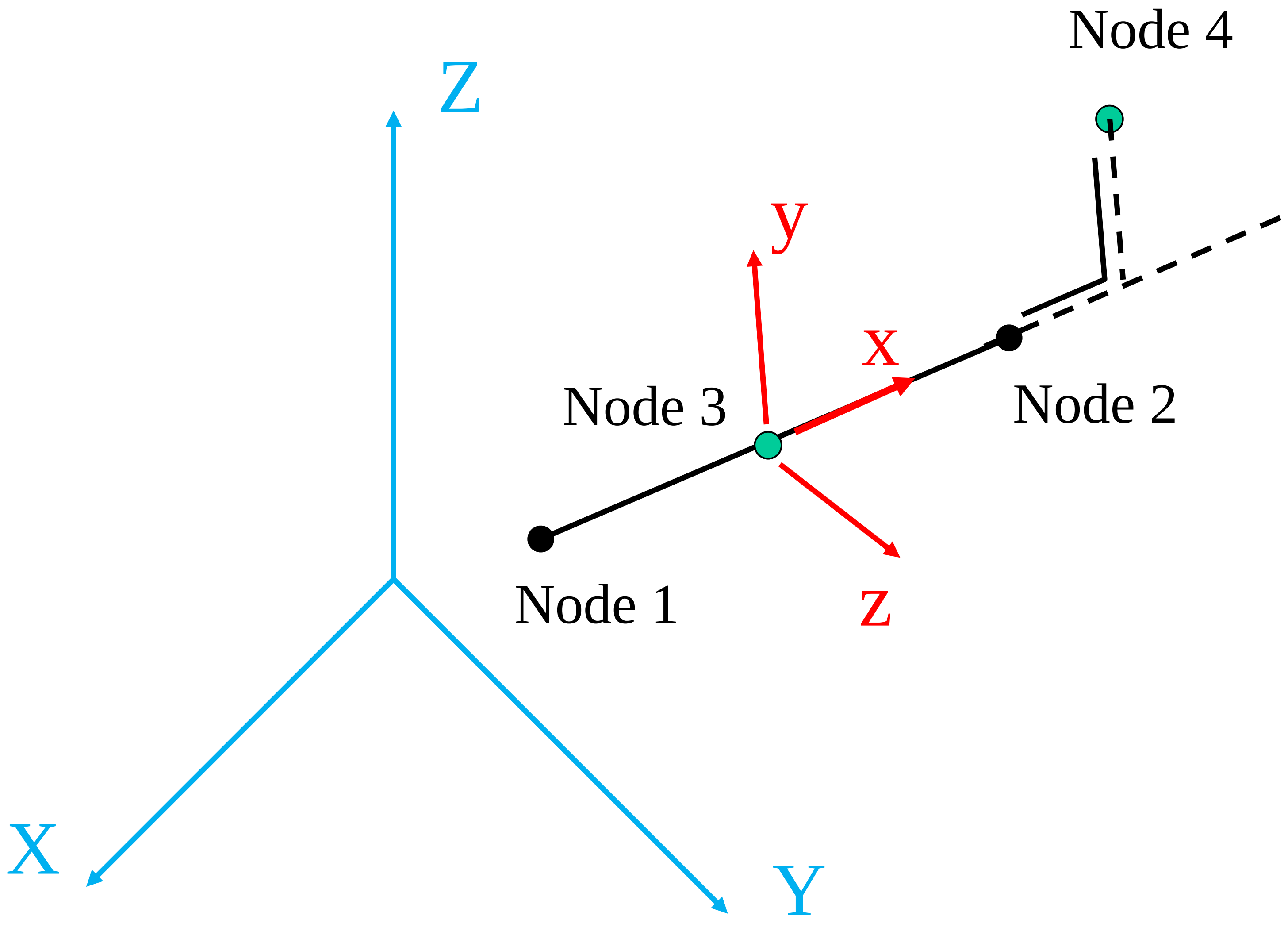
This would not be the case if the elements in the right hand column would have their nodes from bottom to top.

3D beam finite element



In which direction is pointing y?

3D beam finite element



y is in the plane defined by nodes 1, 2 and 4,
in the direction from axis x to node 4.

1) Local system of coordinates

2) Degrees of freedom

3) The basic equations

4) Material properties

5) Numerical integration on the length

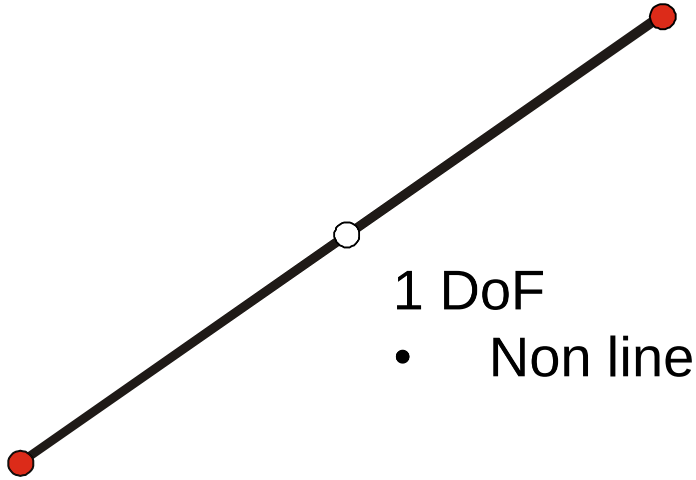
6) Numerical integration on the surface

7) Stiffness in torsion

8) Examples

One 3D beam finite element

- Prismatic.
- Straight in the initial configuration.



7 DoF

- 3 displacements
- 3 rotations
- 1 warping

1 DoF

- Non linear part of the axial displacement

7 DoF

- 3 displacements
- 3 rotations
- 1 warping

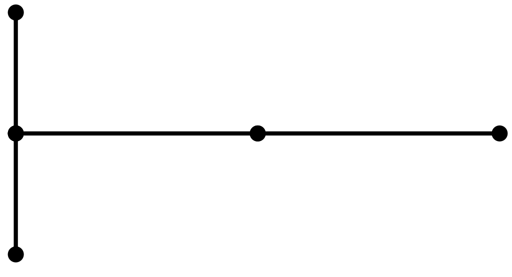
Transmission of warping between adjacent elements?

✓ In continuous elements: yes

✓ At joints: no

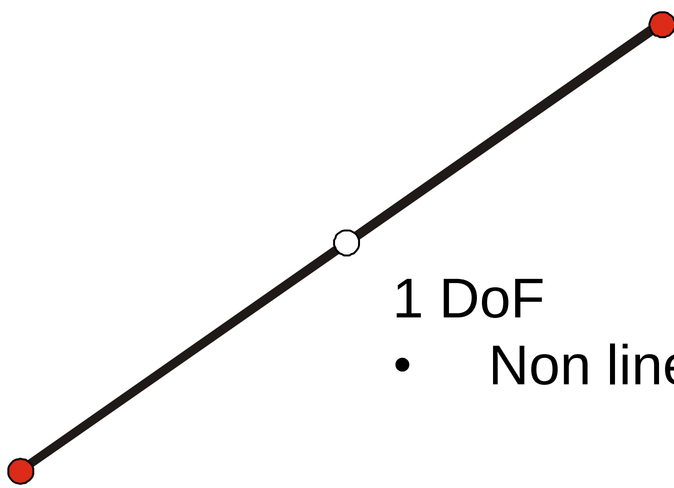
⇒ either fix the warping to 0 at the common node.

⇒ or use the RELAX command to release the warping (w = 0) at the end of some elements.



One 2D beam finite element

- ➢ Prismatic.
- ➢ Straight in the initial configuration.



3 DoF

- ▪ 2 displacements
- ▪ 1 rotation

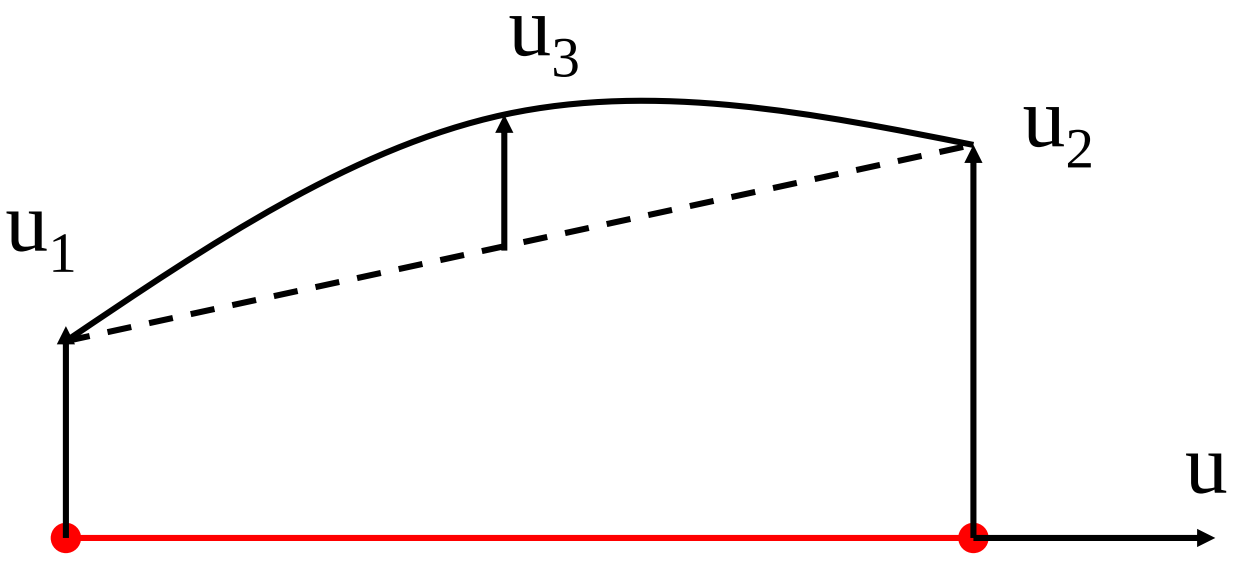
1 DoF

- • Non linear part of the axial displacement

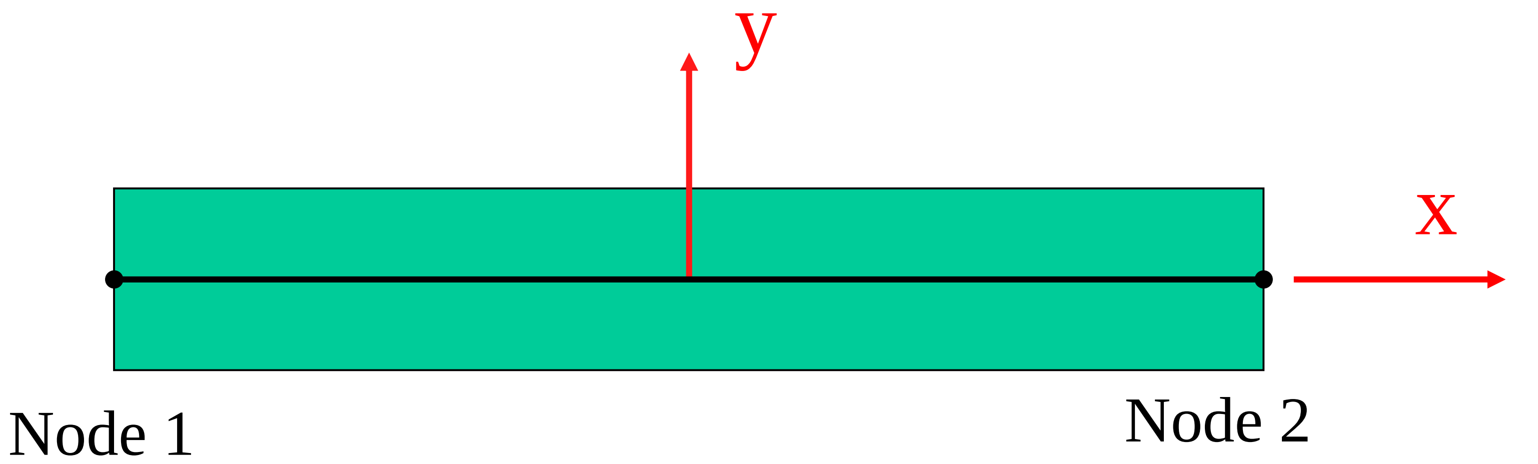
3 DoF

- ▪ 2 displacements
- ▪ 1 rotation

The central node bears the non linear component of the axial displacement.



Why ????

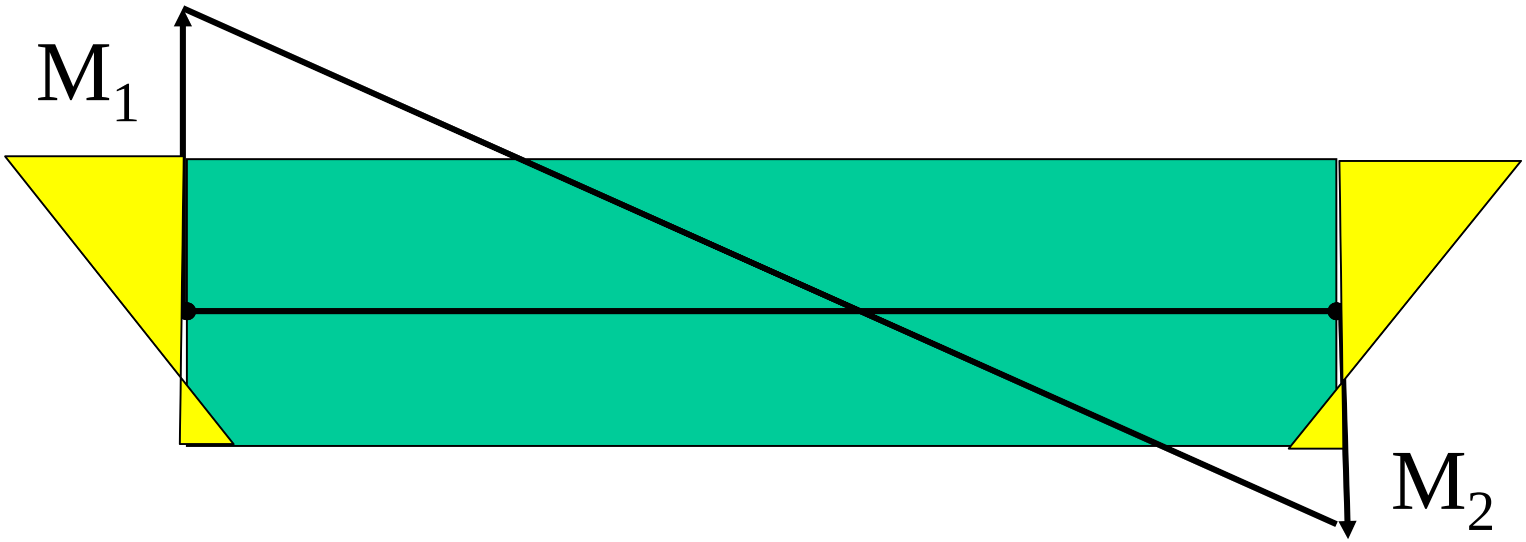


With only 2 nodes, the axial displacement *u* of the node line can only be a linear function of x.

$\Rightarrow$ On the node line (y = 0), the strain d*u*/dx can only be constant.

$\Rightarrow$ This leads to too stiff elements.

Example: linear bending moment in a section that is not symmetrical.
The strain at mid-level is not constant, but a beam F.E. with 2 nodes cannot represent this.

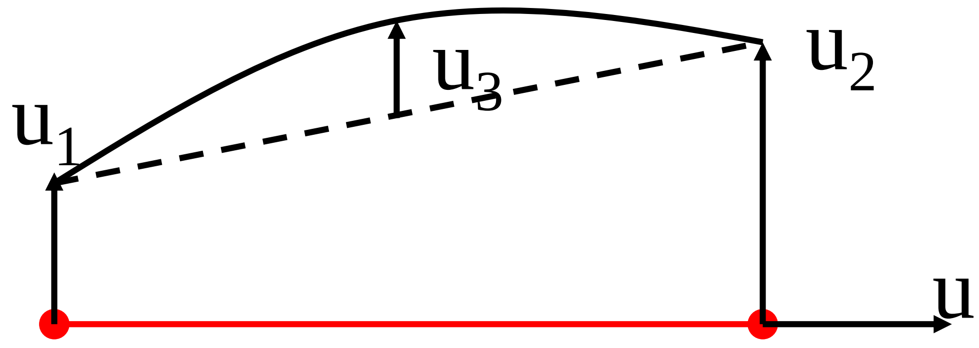


It is desirable to have a strain that varies along the node line.
This is possible if the axial displacement is f(x^2).
An additional DoF is needed. It is supported by the central node.

Note that the effect of the central node has nothing to do with:

- Large displacements
- Effects of fire

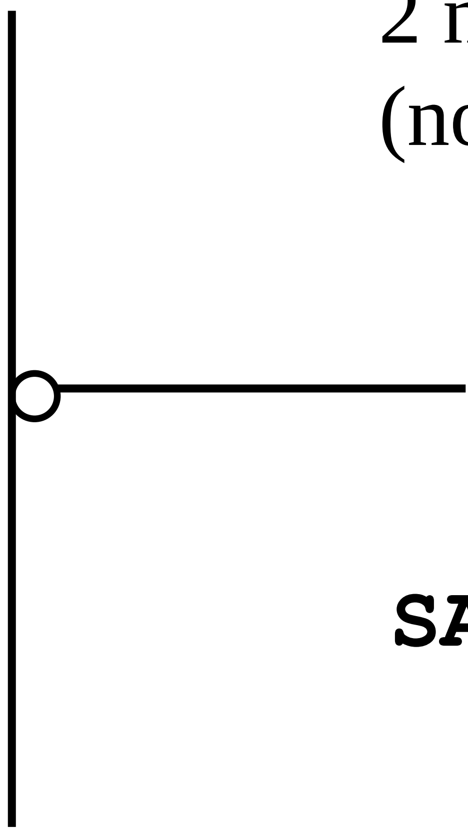
This central node is not required in elastic elements (the strain is not used).



Elements in which u_3 is 0 during the simulation don't have much plasticity.
Elements in which u_3 starts to increase during the simulation are those where a lot of plasticity develops

Internal hinges

First solution used historically



2 nodes at the same location
(node i and node j)

```
SAME   i   j   YES   YES   NO
```

X Y θ

This solution has 2 drawbacks:

1) The linked DoF’s are parallel to the global system of coordinates
   => Not applicable for diagonals in 3D models

1) The linked DoF’s don’t follow the elements in large displacements
   => Approximations in 3D structures

New solution = Relaxation of the force(s) at the end of the beam elements.

- One or several internal forces (M, N, V…) are forced to be equal to 0 at the end of the elements.

- It is also possible to introduce a semi-rigid connection for each DoF.

```
RELAX_ELEM  ! Start of the section on relaxations
  BEAMS     ! Only beam elements can be relaxed
      ELEM   1     -1 -1 -1  -1  0  0  -1      -1 -1 -1  -1 -1 -1  -1
      ELEM  10     -1 -1 -1  -1 -1 -1  -1      -1 -1 -1  -1  0  0  -1
  END_BEAMS ! End of the beams
END_RELAX   ! End of the section
```

Element number

Node 1

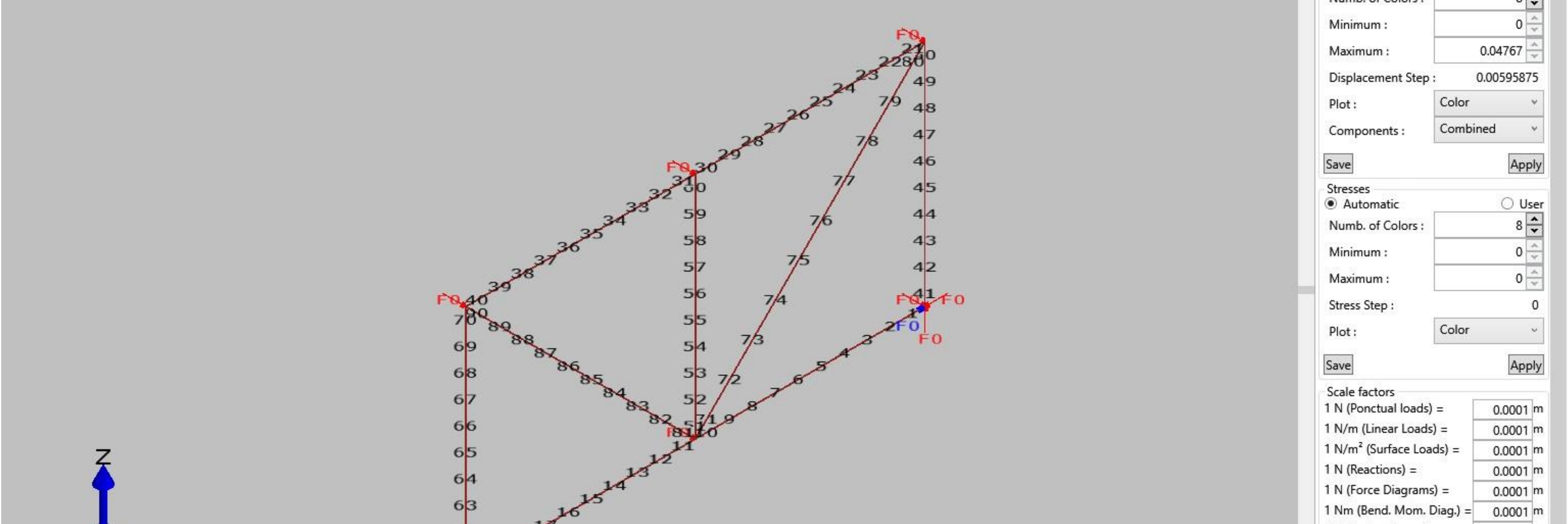
Node 2

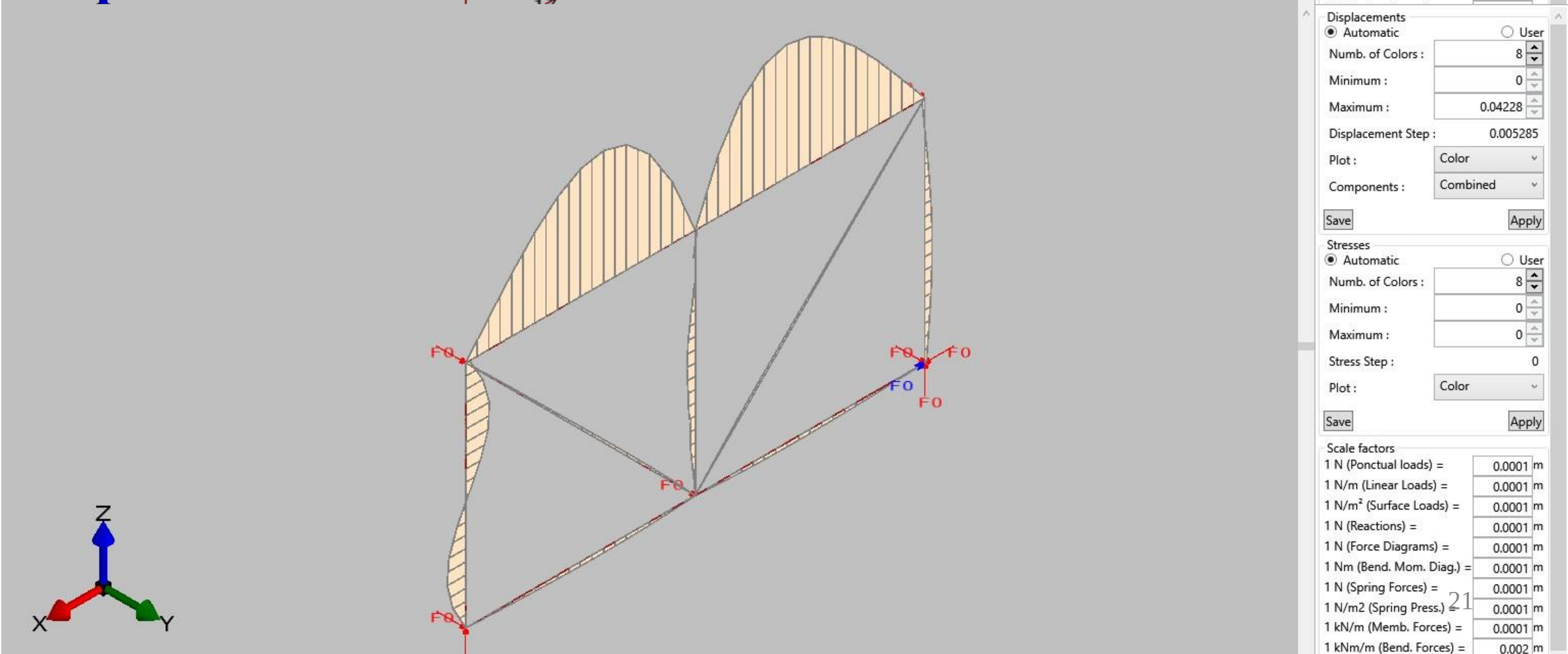
The structural member made of elements 1, 2,… 9, 10 has no bending moment with respect to local axes y and z at its ends:

Negative value means: No relaxation

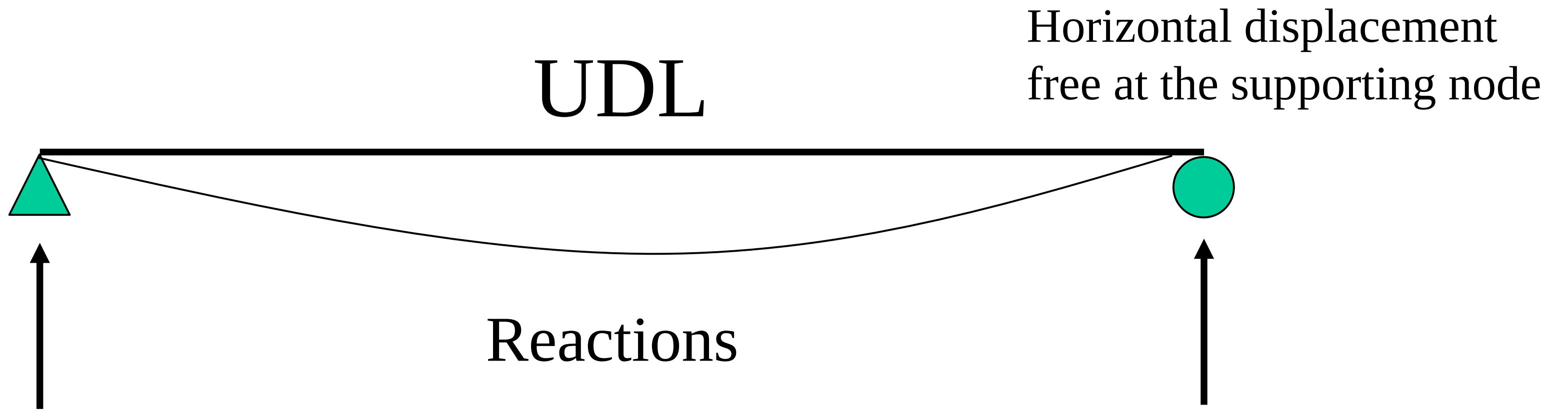
Zero value means: Complete relaxation

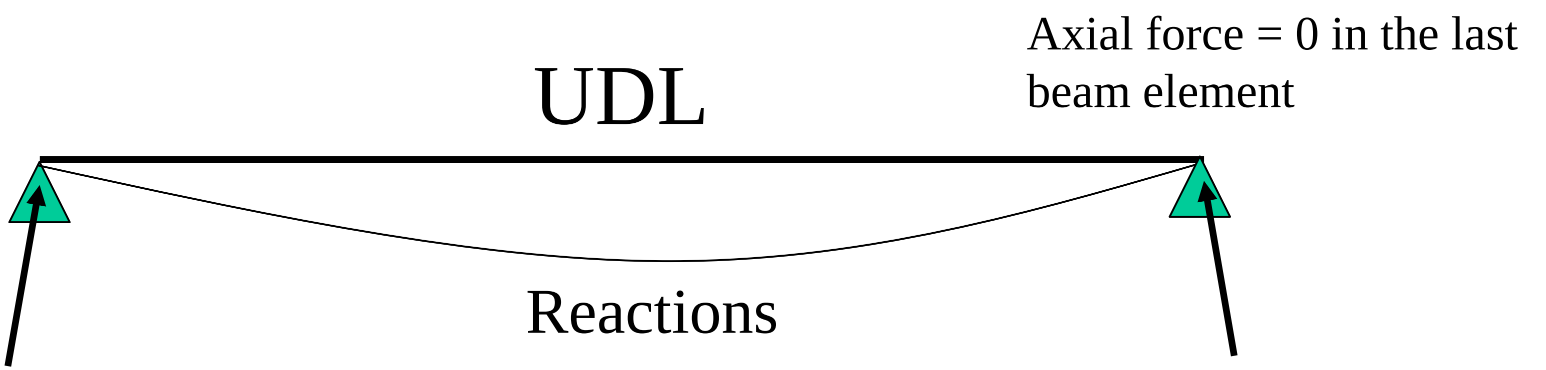
Positive value means: Stiffness of the semi-rigid connection (in N/m or in Nm/rad)


Minimum : 0
Maximum : 0.04767
Displacement Step : 0.00595875
Plot : Color
Components : Combined
Save
Apply
Stresses
Automatic
User
Numb. of Colors : 8
Minimum : 0
Maximum : 0
Stress Step : 0
Plot : Color
Save
Apply
Scale factors
1 N (Ponctual loads) = 0.0001 m
1 N/m (Linear Loads) = 0.0001 m
1 N/m² (Surface Loads) = 0.0001 m
1 N (Reactions) = 0.0001 m
1 N (Force Diagrams) = 0.0001 m
1 Nm (Bend. Mom. Diag.) = 0.0001 m
Z


Displacements
Automatic
User
Numb. of Colors : 8
Minimum : 0
Maximum : 0.04228
Displacement Step : 0.005285
Plot : Color
Components : Combined
Save
Apply
Stresses
Automatic
User
Numb. of Colors : 8
Minimum : 0
Maximum : 0
Stress Step : 0
Plot : Color
Save
Apply
Scale factors
1 N (Ponctual loads) = 0.0001 m
1 N/m (Linear Loads) = 0.0001 m
1 N/m² (Surface Loads) = 0.0001 m
1 N (Reactions) = 0.0001 m
1 N (Force Diagrams) = 0.0001 m
1 Nm (Bend. Mom. Diag.) = 0.0001 m
1 N (Spring Forces) = 0.0001 m
1 N/m2 (Spring Press.) = 0.0001 m
1 kN/m (Memb. Forces) = 0.0001 m
1 kNm/m (Bend. Forces) = 0.002 m
Z
X
Y

! The relaxations are in the **local** System of Coordinates of the element





1) Local system of coordinates

2) Degrees of freedom

3) The basic equations

4) Material properties

5) Numerical integration on the length

6) Numerical integration on the surface

7) Stiffness in torsion

8) Examples

The local equilibrium equation is :

$$\frac{\partial \sigma_x}{\partial y} + \frac{\partial \sigma_y}{\partial x} \ldots\ldots\ldots$$

The element equilibrium equation is :

$$[K]\{u\} \quad = \quad \{P\}$$

$$\begin{bmatrix} k_{1,1} & k_{1,2} & k_{1,3} & & & & k_{1,7} \\ & k_{2,2} & & & & & \\ & & k_{3,3} & & & & \\ & & & k_{4,4} & & & \\ & & & & k_{5,5} & & \\ & & & & & k_{6,6} & k_{6,7} \\ Sym & & & & & & k_{7,7} \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ \\ \\ \\ u_6 \\ u_7 \end{Bmatrix} = \begin{Bmatrix} p_1 \\ p_2 \\ \\ \\ \\ p_6 \\ p_7 \end{Bmatrix}$$

1) Local system of coordinates

2) Degrees of freedom

3) The basic equations

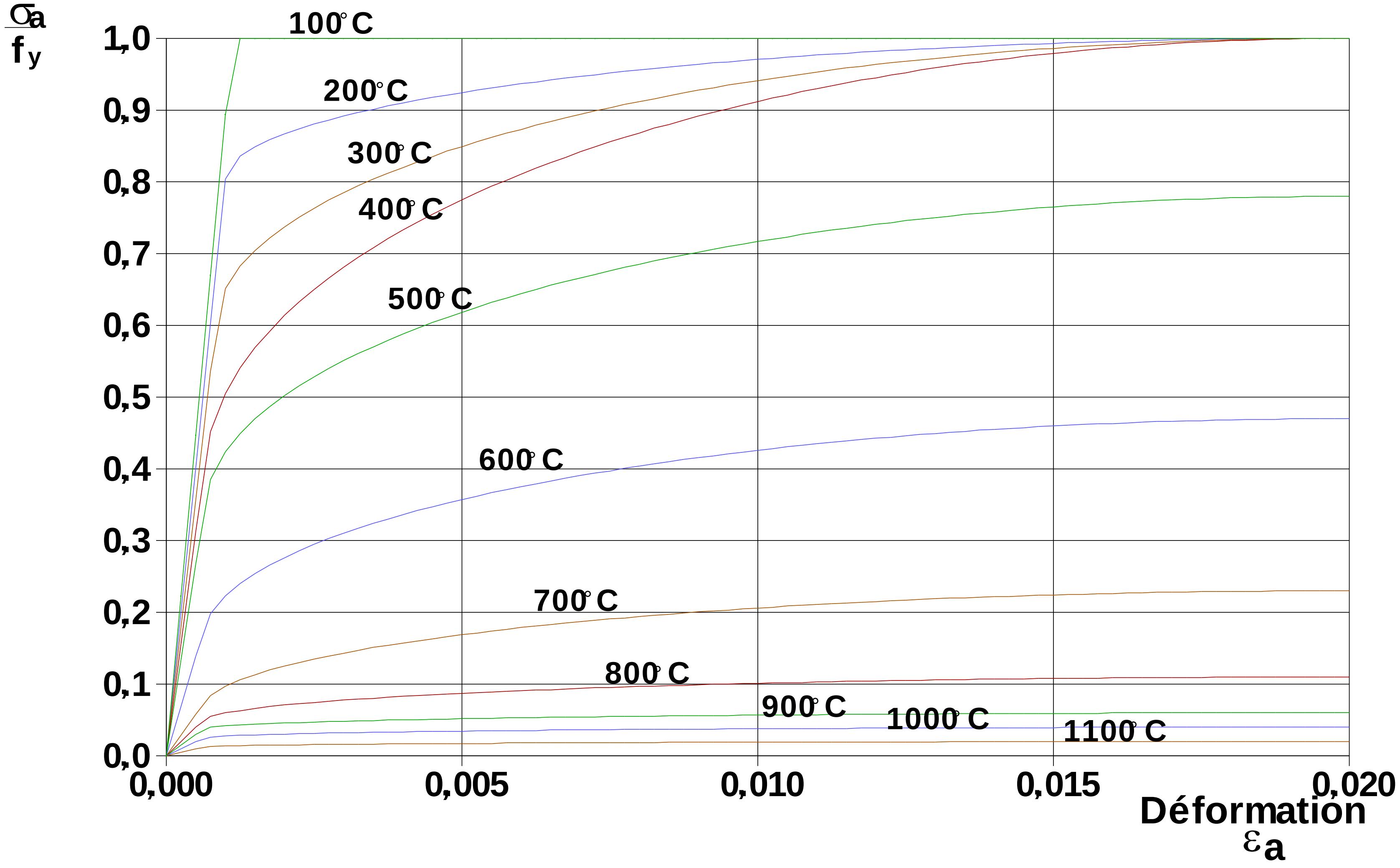
4) Material properties

5) Numerical integration on the length

6) Numerical integration on the surface

7) Stiffness in torsion

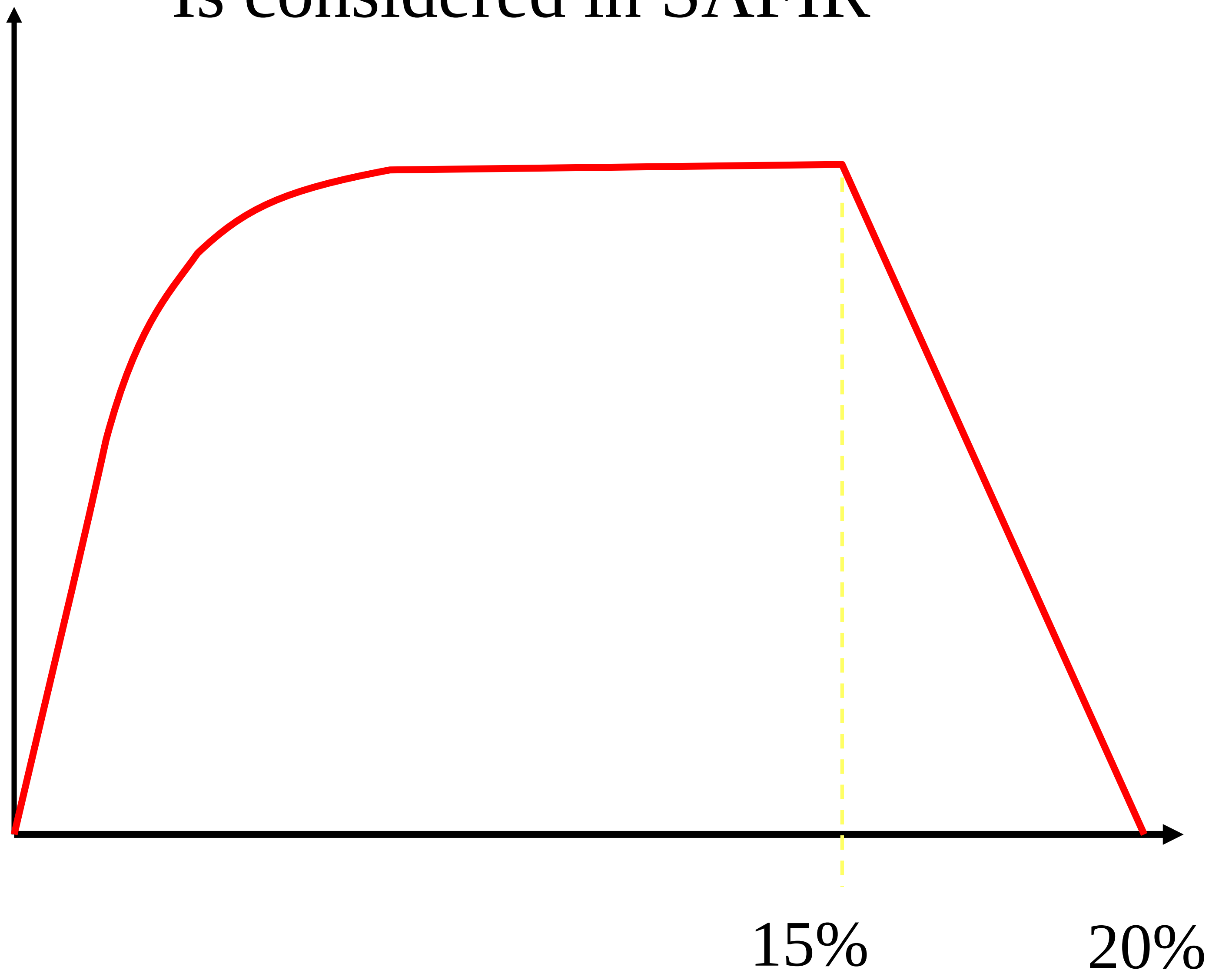
8) Examples

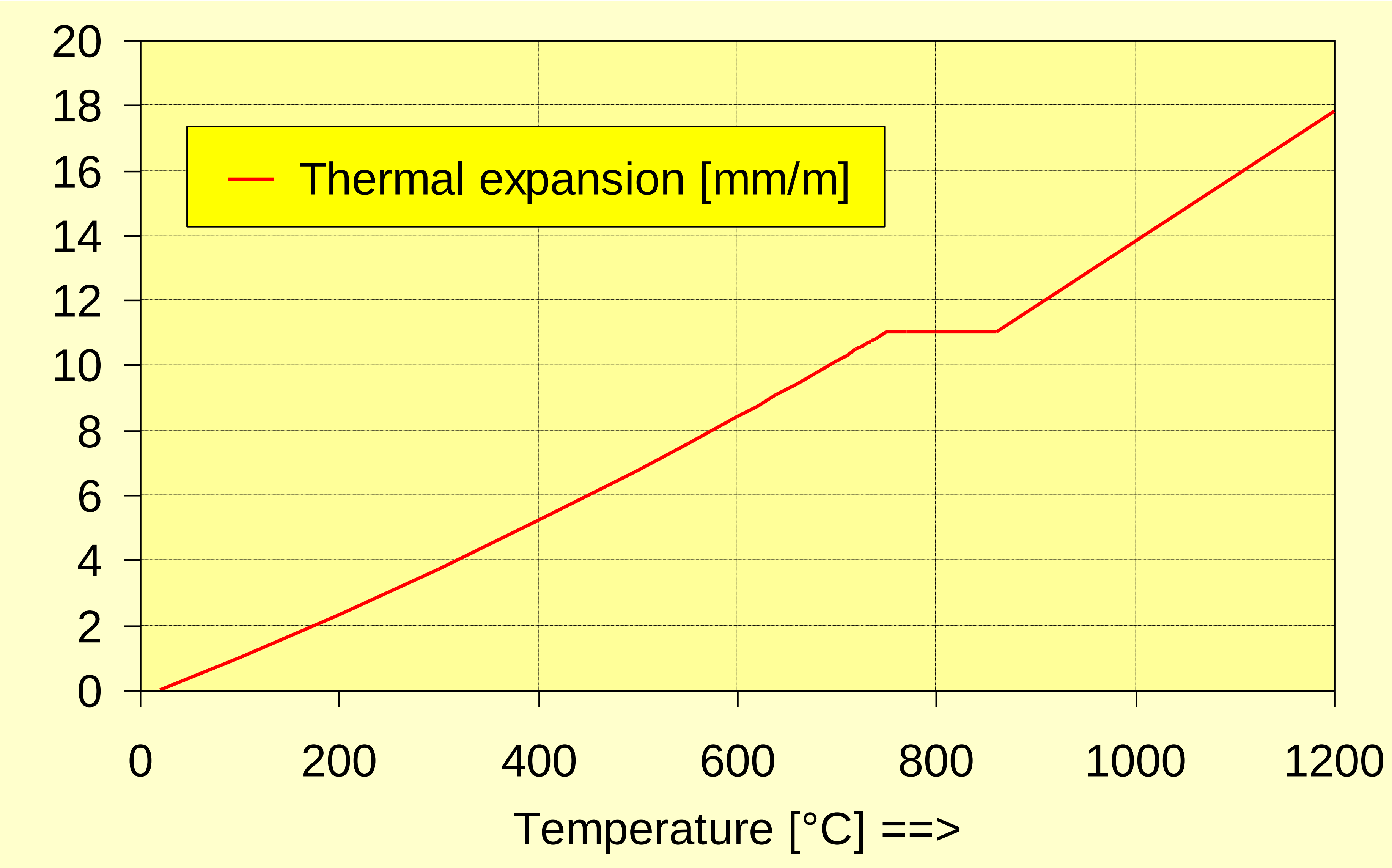


Stress-strain relationship in steel

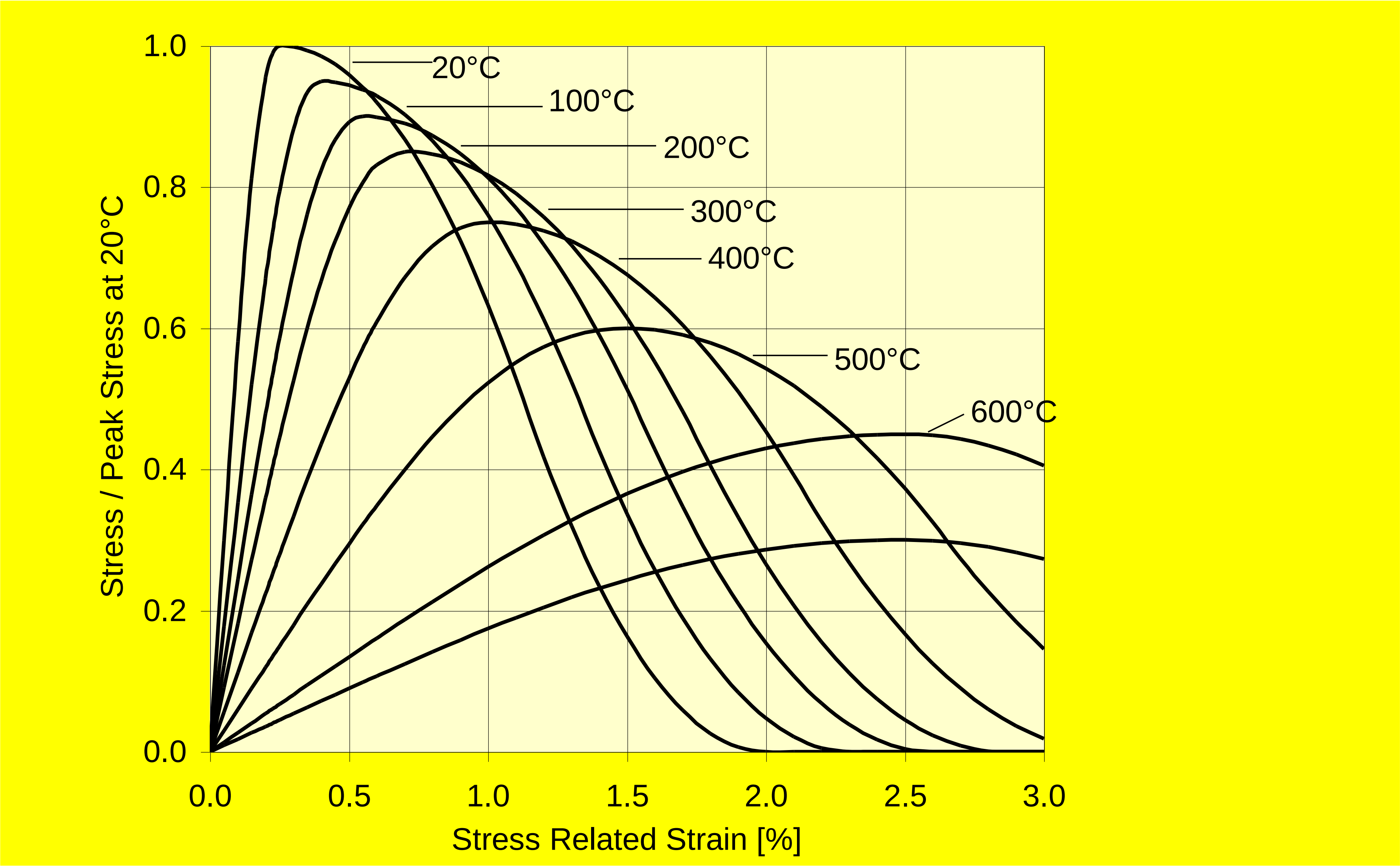
Descending branch.

Is considered in SAFIR

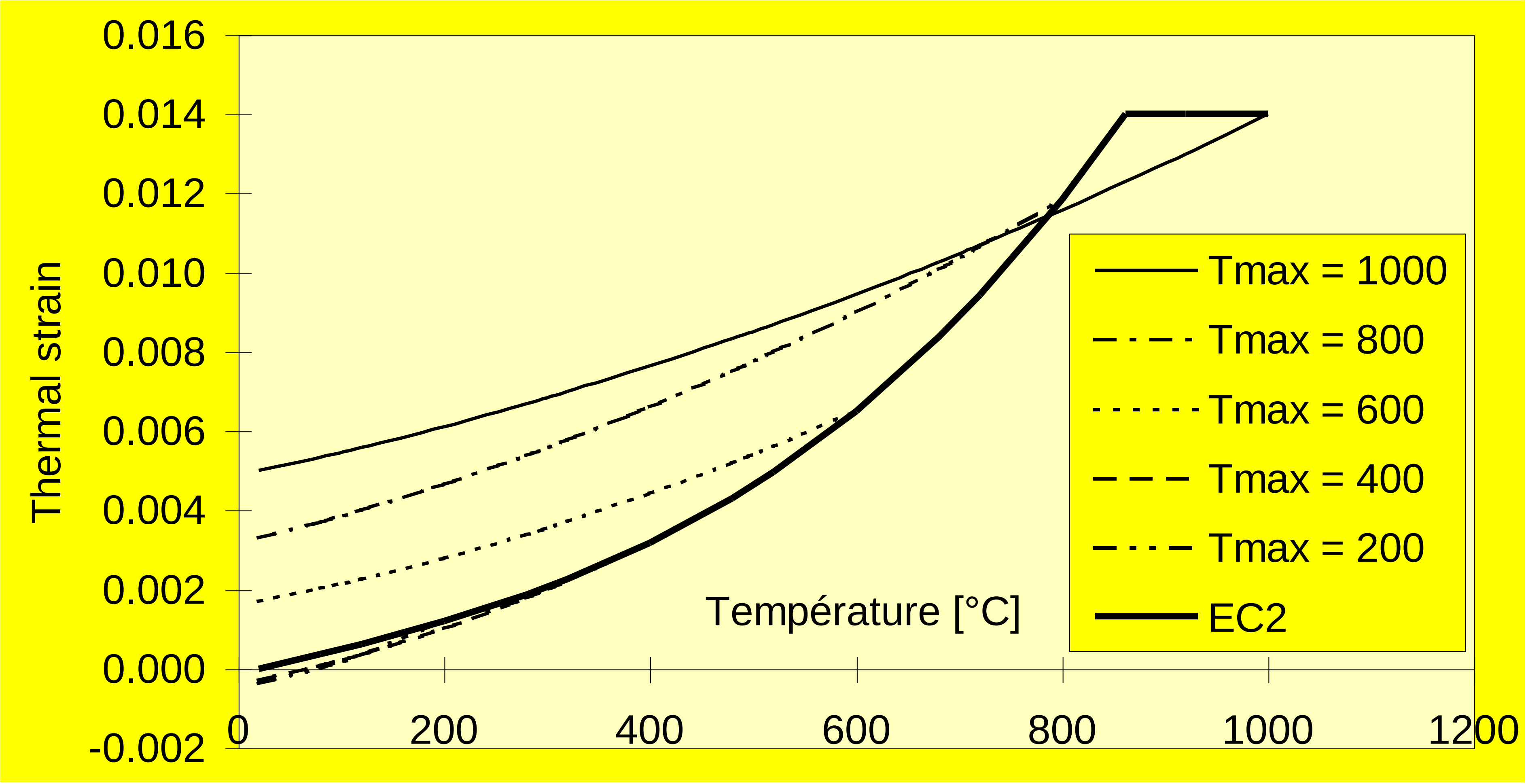




Thermal expansion in steel



Stress-Strain relationship in concrete



Thermal expansion of concrete
Non reversible during cooling

TRANSLATE ilocal iglobal

Section 1
Material 1 (steel)
Material 2 (concrete)

Section 2
Material 1 (concrete)
Material 2 (timber)

TRANSLATE 1 1
TRANSLATE 2 3

TRANSLATE 1 3
TRANSLATE 2 2

Structure
Material 1 (steel)
Material 2 (timber)
Material 3 (concrete)

1) Local system of coordinates

2) Degrees of freedom

3) The basic equations

4) Material properties

5) Numerical integration on the length

6) Numerical integration on the surface

7) Stiffness in torsion

8) Examples

$$Nodal\ \ forces = \int_V \sigma\ f\left(x, y, z\right) dV$$

$$Stiffness \qquad = \int_V E\ f\left(x, y, z\right) dV$$

$$Nodal\ \ forces = \int_V \sigma\ f\left(x,y,z\right)dV$$

$$= \int_0^L \int_S \sigma\ f\left(x,y,z\right)dS\,dL$$

$$Stiffness \qquad = \int_V E\ f\left(x,y,z\right)dV$$

$$= \int_0^L \int_S E\ f\left(x,y,z\right)dS\,dL$$

$$Nodal\ \ forces = \int_V \sigma\ f\left(x,y,z\right) dV$$

$$= \int_0^L \int_S \sigma\ f\left(x,y,z\right) dS\, dL$$

$$= \sum_{i=1}^{NG} \omega_i \int_S \sigma\ f\left(x,y,z\right) dS$$

$$Stiffness \qquad = \int_V E\ f\left(x,y,z\right) dV$$

$$= \int_0^L \int_S E\ f\left(x,y,z\right) dS\, dL$$

$$= \sum_{i=1}^{NG} \omega_i \int_S E\ f\left(x,y,z\right) dS$$

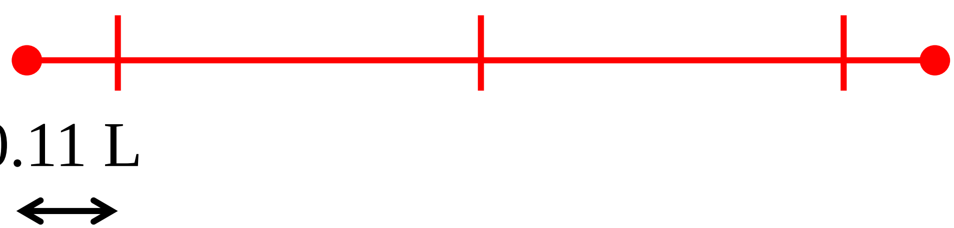
Integrations on the surface are made NG times, using the discretisation of the thermal analysis

Results at the Gausian integration points for a 3D beam:

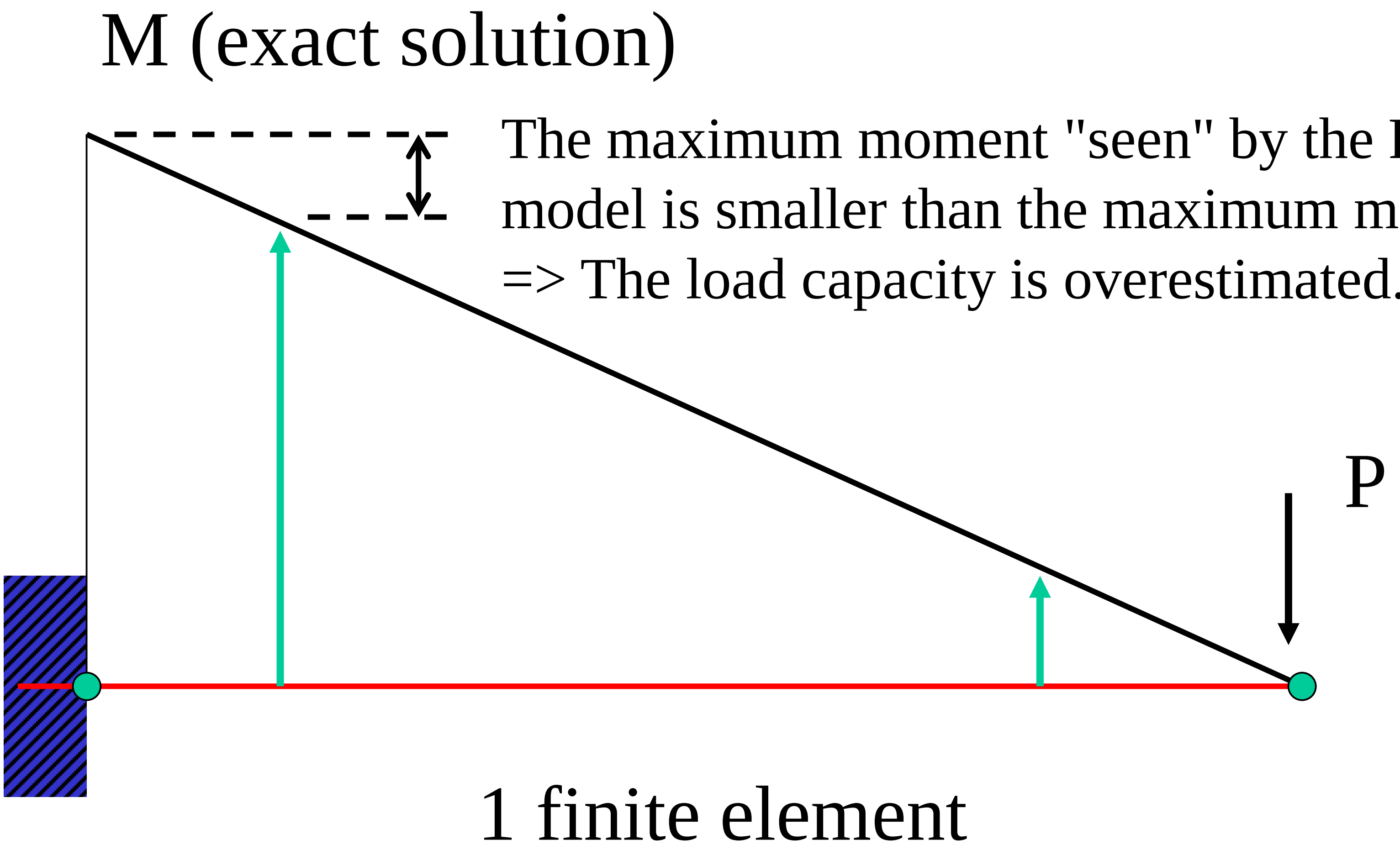
- N, M_y, M_z, M_t, V_y, V_z
- EA, ES, EI (`PRNEIBEAM`)
- + at each fiber: σ, E_t, ε_m (optional)

NG = 2

NG = 3

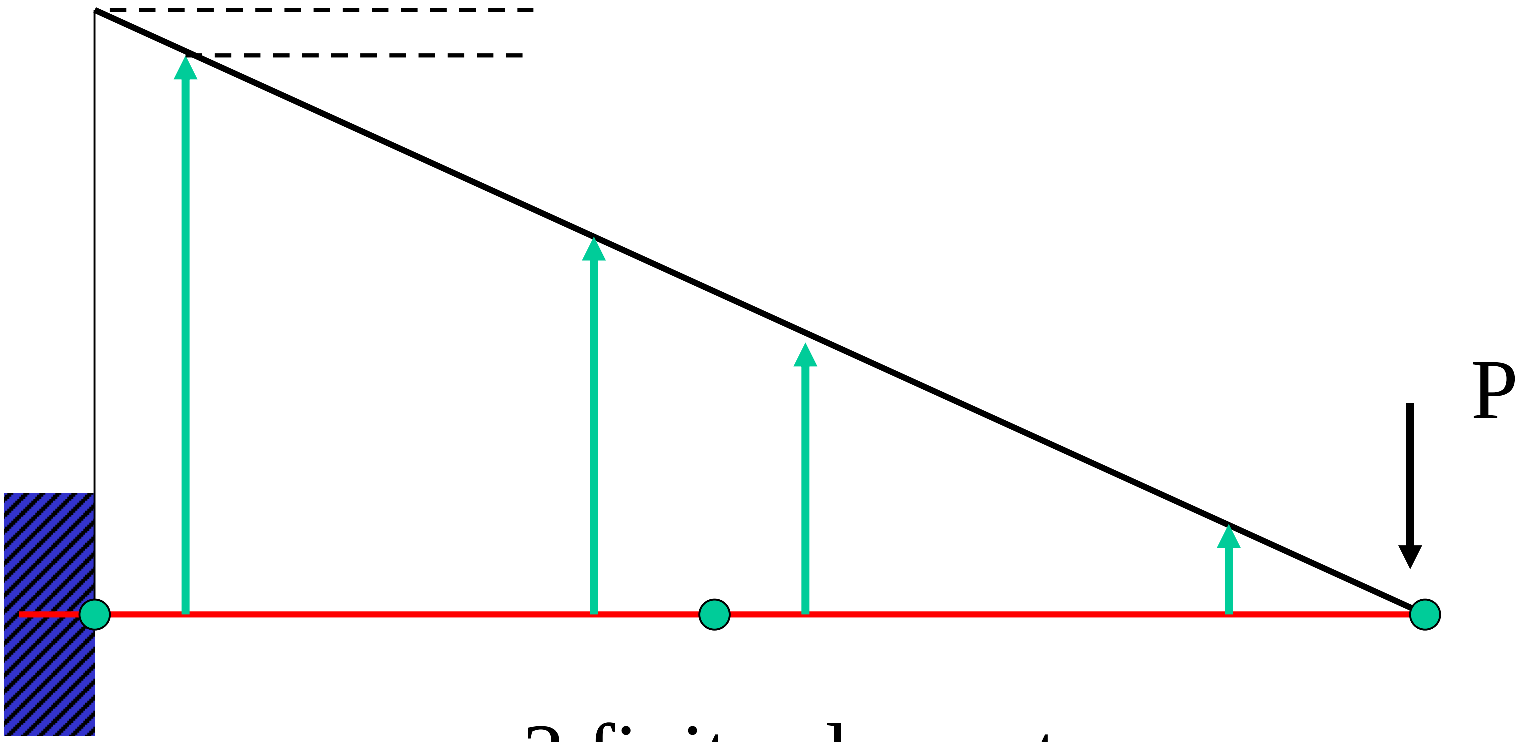


No point of integration at the ends of the element: consequence

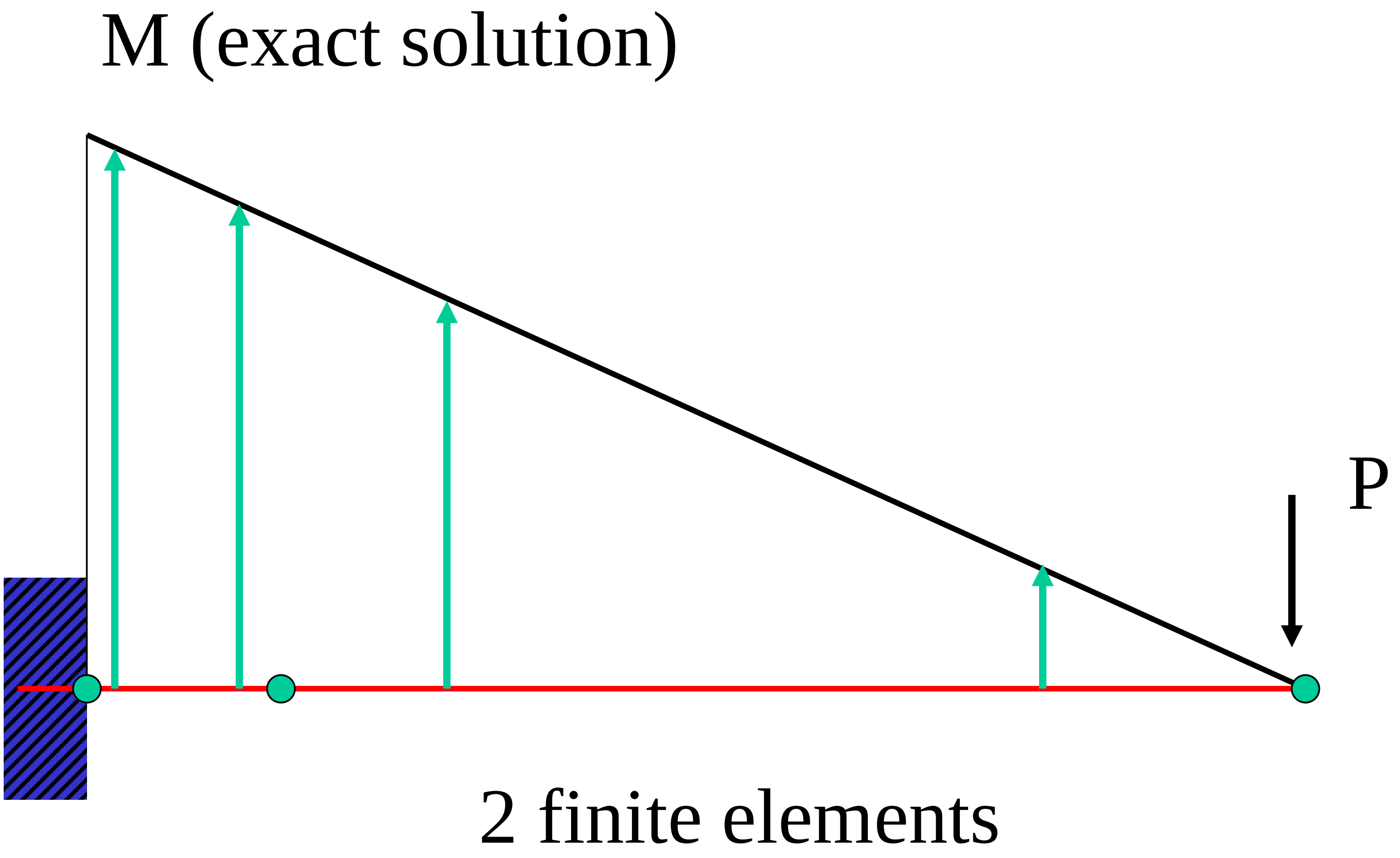


The maximum moment "seen" by the F.E. model is smaller than the maximum moment => The load capacity is overestimated.

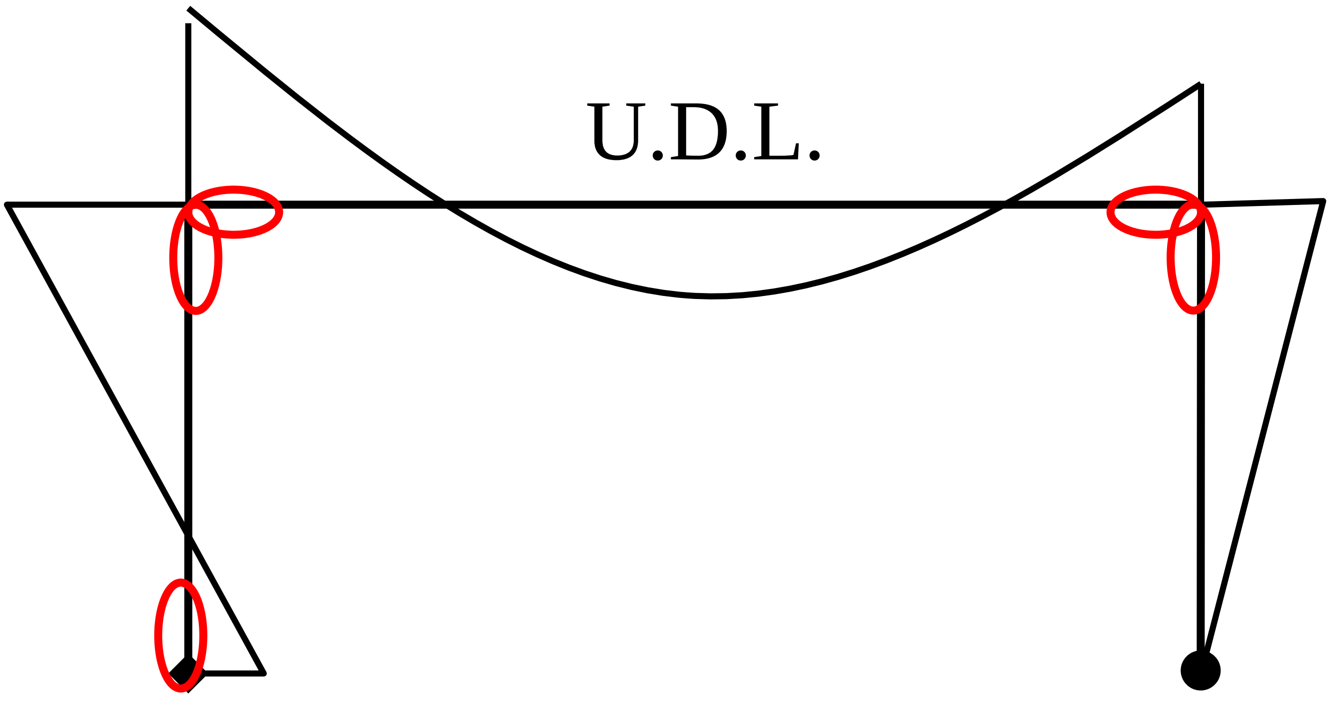
M (exact solution)



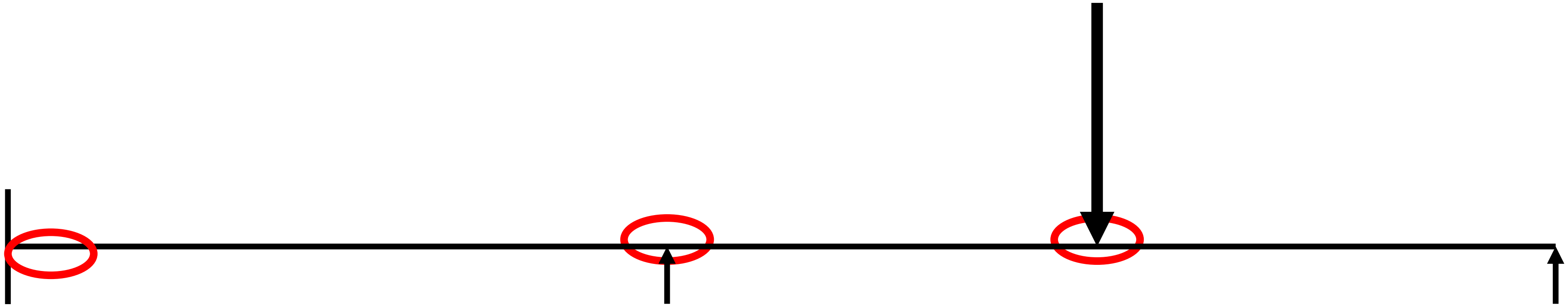
2 finite elements


M (exact solution)
P
2 finite elements

Conclusions: Beam F.E. not too long in the zones of peaks (= maximum or minimum) in the bending moment diagram



Conclusions: Beam F.E. not too long in the zones of peaks in the bending moment diagram

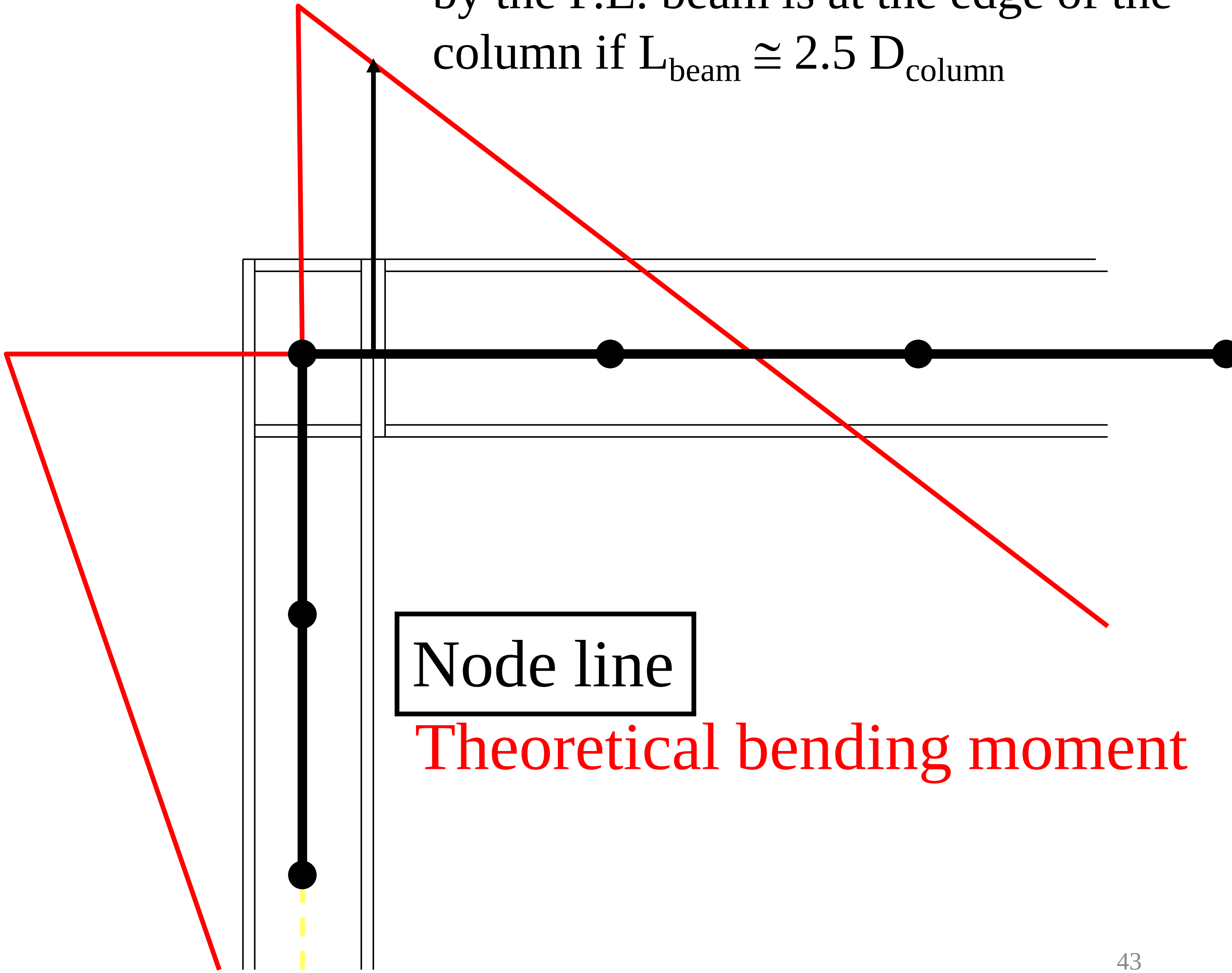


Conclusions: Beam F.E. not too long in the zones of peaks in the bending moment diagram

How long is too long ?

$$L_{beam} \geq D_{beam}$$

Position of the biggest moment seen by the F.E. beam is at the edge of the column if $L_{beam} \cong 2.5\ D_{column}$



1) Local system of coordinates

2) Degrees of freedom

3) The basic equations

4) Material properties

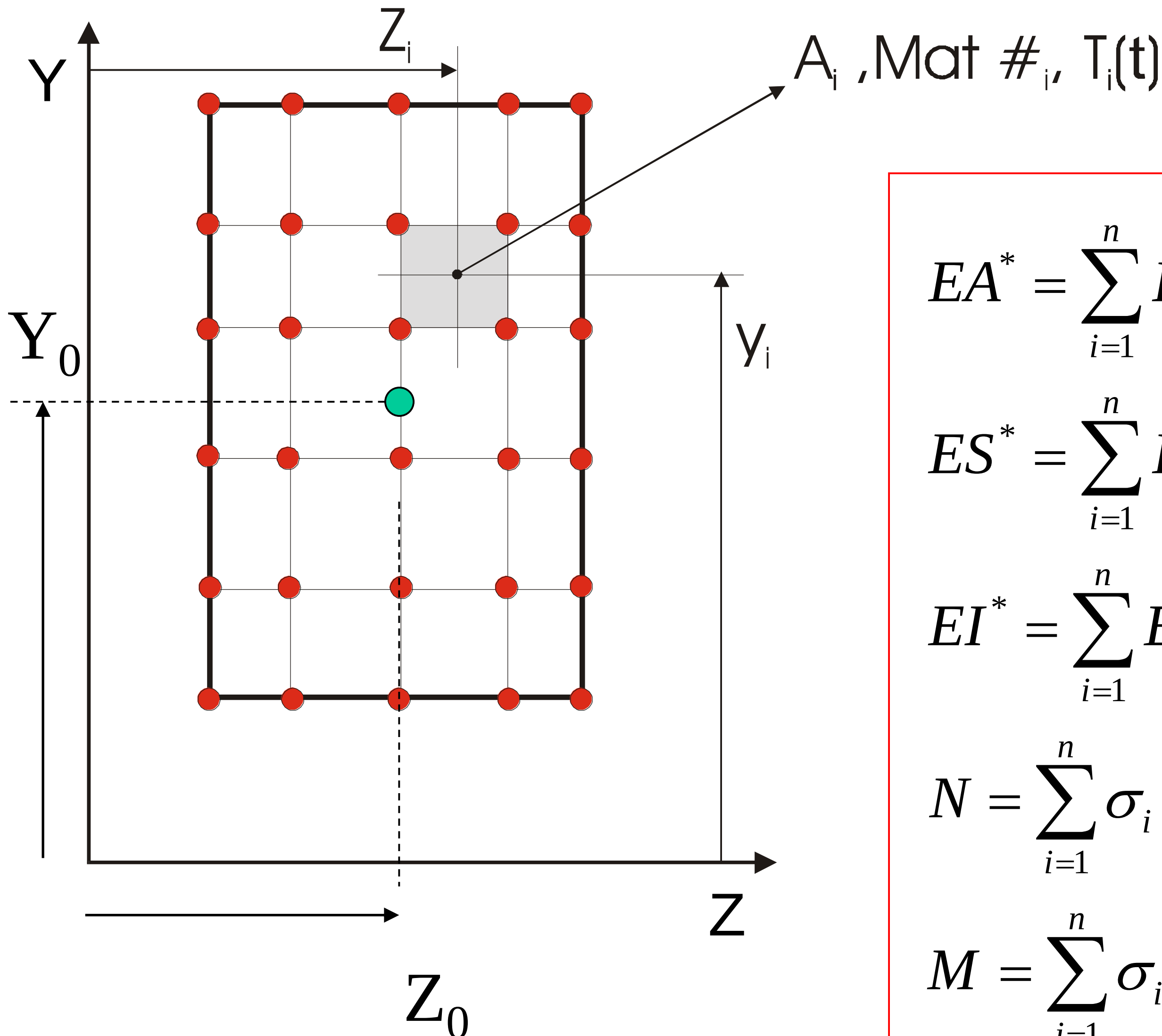
5) Numerical integration on the length

6) Numerical integration on the surface

7) Stiffness in torsion

8) Examples

Integration on the section of the beam element



$$EA^* = \sum_{i=1}^{n} E_i\left(T_i\right) \times A_i$$

$$ES^* = \sum_{i=1}^{n} E_i\left(T_i\right) \times \left(y_i - y_0\right) A_i$$

$$EI^* = \sum_{i=1}^{n} E_i\left(T_i\right) \times \left(y_i - y_0\right)^2 A_i$$

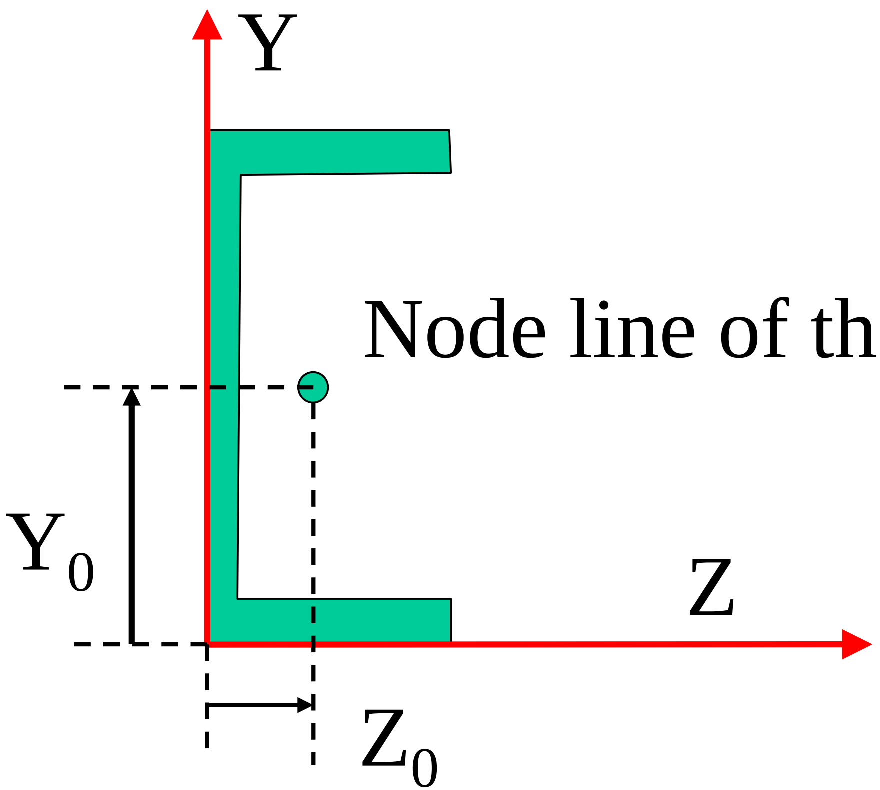
$$N = \sum_{i=1}^{n} \sigma_i\left(T_i\right) \times A_i$$

$$M = \sum_{i=1}^{n} \sigma_i\left(T_i\right) \times \left(y_i - y_0\right) A_i$$

Structure of the input file for thermal analyses

NODELINE Y_0 Z_0

YC_ZC Y_c Z_c



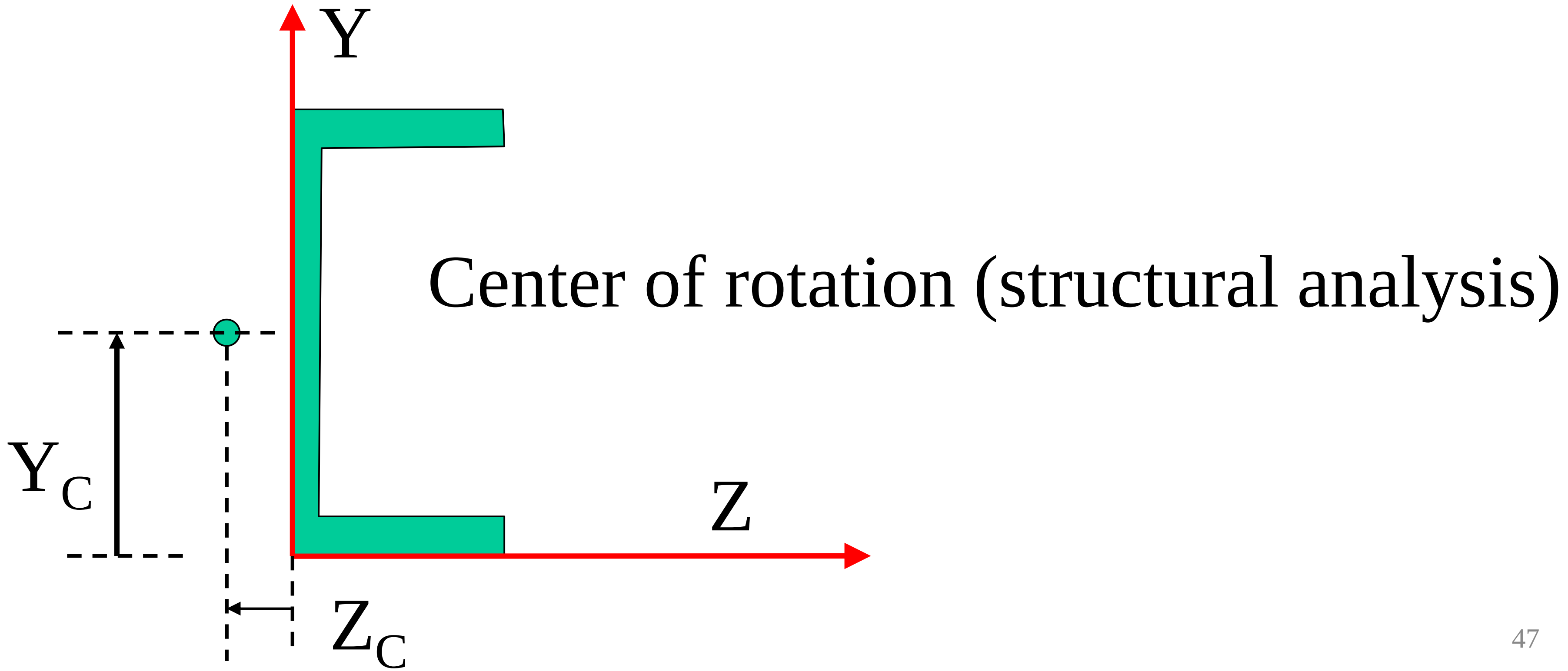
Node line of the structural analysis

Sytem of coordinates.
Chosen to allow easy introduction of the node coordinates

Structure of the input file for thermal analyses

`NODELINE` Y_0 Z_0

`YC_ZC` Y_c Z_c

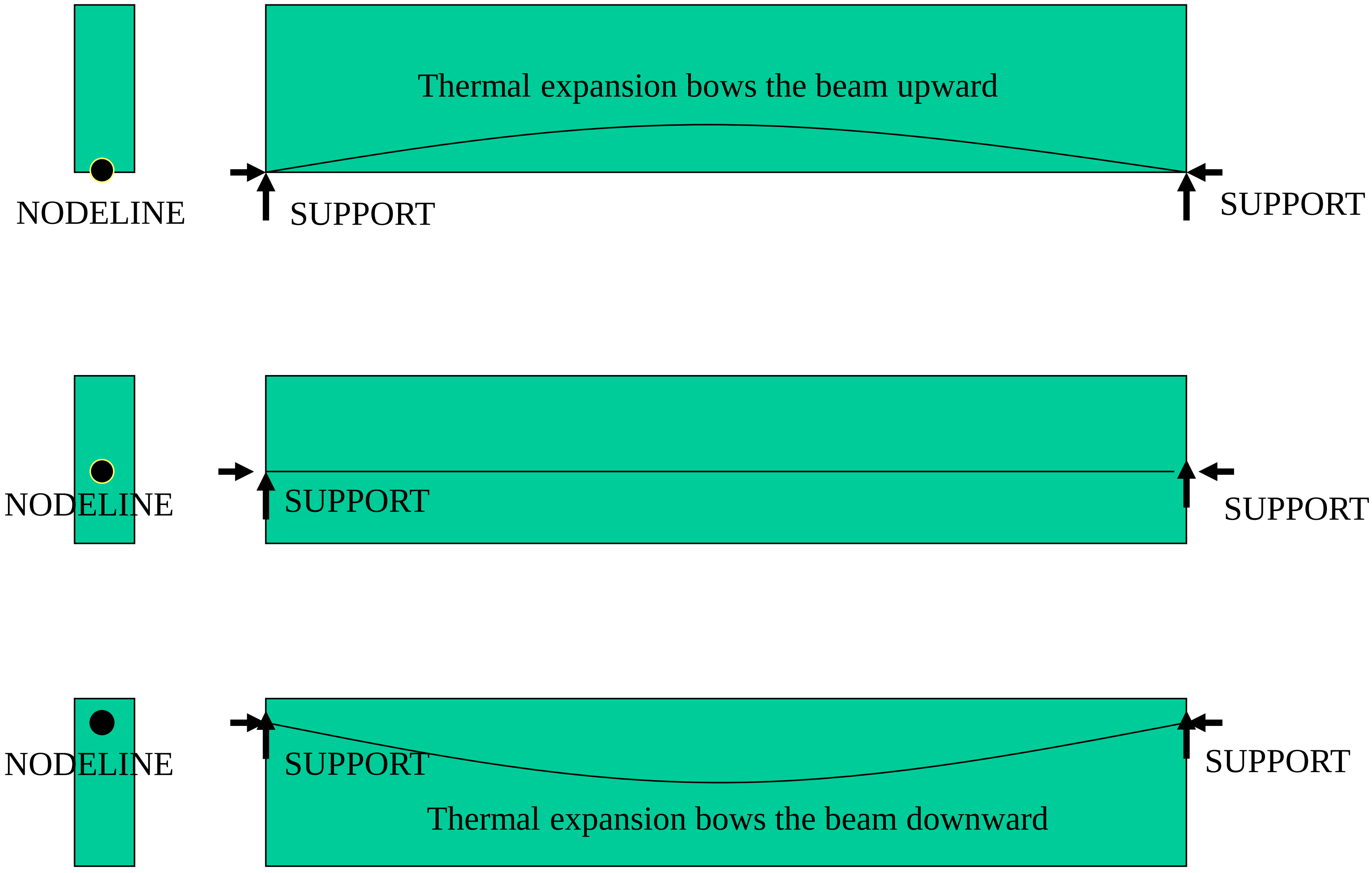


The beam elements are connected in the mechanical analysis by their end nodes, which are located on the nodeline in the section.

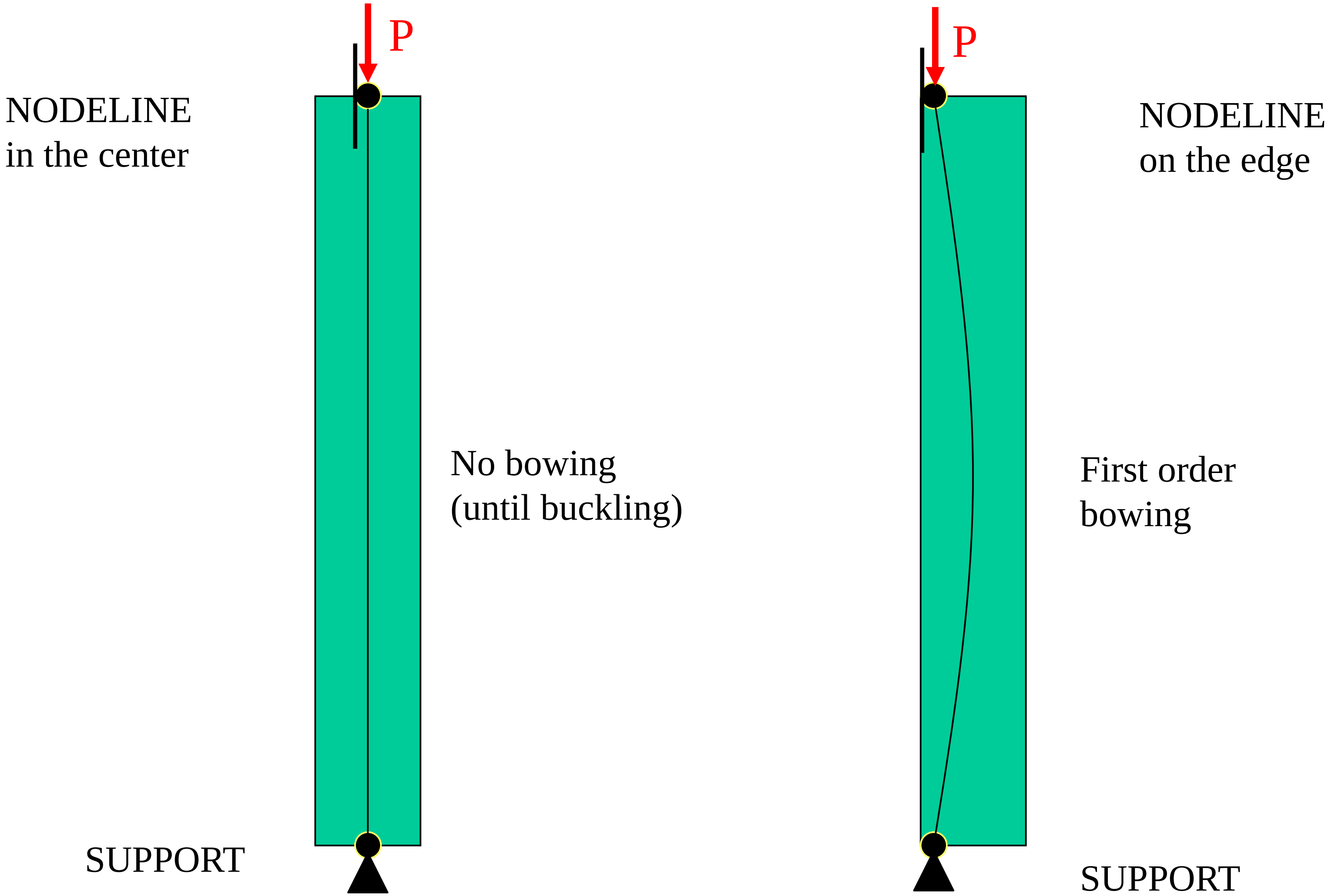
Concentrated forces and restrained Degrees of Freedom are applied to the nodes, hence at the node line.

The bending moments are also calculated and given with respect to this node line.

NODELINE Y0 Z0

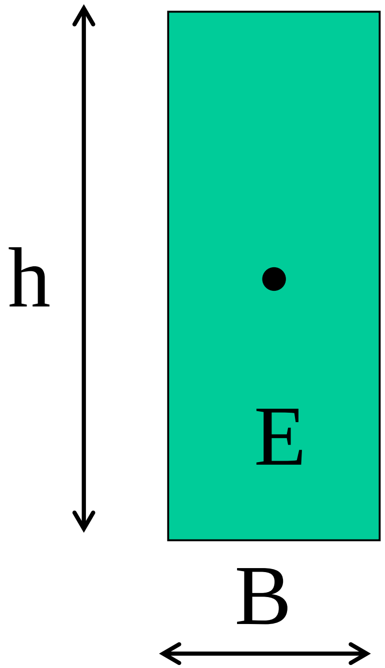


NODELINE Y0 Z0



First order moments = 0 in both cases

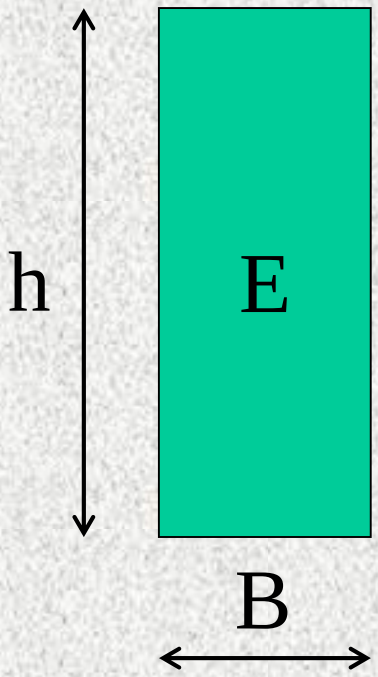
Integration on the section of the beam element



$$EA^* = \sum_{i=1}^{n} E_i\left(T_i\right) \times A_i \qquad = \mathrm{EBh}$$

$$EI^* = \sum_{i=1}^{n} E_i\left(T_i\right) \times \left(y_i - y_0\right)^2 A_i \qquad \boxed{\neq \mathrm{EBh}^3/12}$$

Integration on the section of the beam element



$$EA^* = \sum_{i=1}^{n} E_i\left(T_i\right) \times A_i \quad = \mathrm{EBh}$$

$$EI^* = \sum_{i=1}^{n} E_i\left(T_i\right) \times \left(y_i - y_0\right)^2 A_i \quad \boxed{\neq \mathrm{EBh}^3/12}$$

The .TEM file

```
NFIBERBEAM  450
    FIBERS
  NODELINE  .200E-01  .000E+00
     YC ZC  .000E+00  .000E+00
  .333500E-02  .333333E-02  .889333E-04    1  .000000E+00
 -.333500E-02  .333333E-02  .889333E-04    1  .000000E+00
  .333500E-02  .100000E-01  .889333E-04    1  .000000E+00
 -.333500E-02  .100000E-01  .889333E-04    1  .000000E+00
  .333500E-02  .166667E-01  .889333E-04    1  .000000E+00
 -.333500E-02  .166667E-01  .889333E-04    1  .000000E+00
  .333500E-02  .233333E-01  .889333E-04    1  .000000E+00
 -.333500E-02  .233333E-01  .889333E-04    1  .000000E+00
```

y_i z_i A_i Mat_i $\sigma_{r,i}$

1) Local system of coordinates

2) Degrees of freedom

3) The basic equations

4) Material properties

5) Numerical integration on the length

6) Numerical integration on the surface

7) Stiffness in torsion

8) Examples

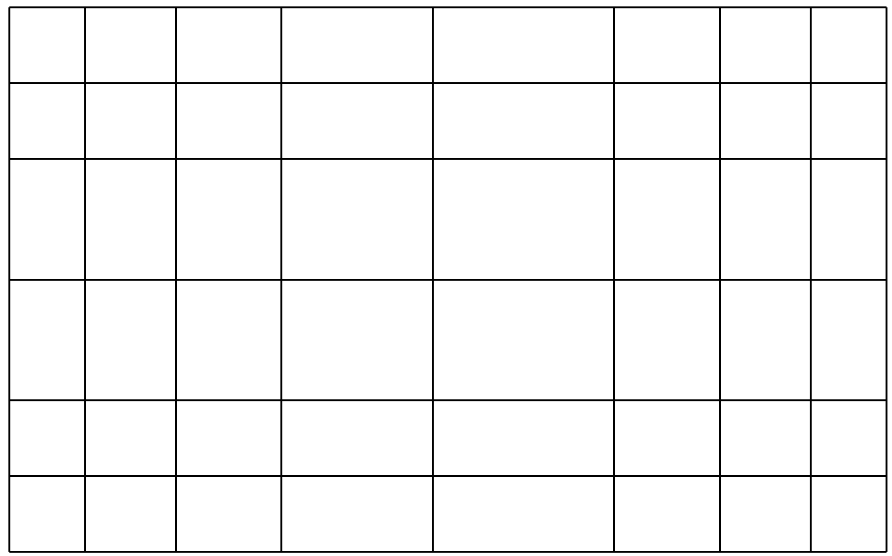
The 3D beam finite element needs the stiffness in torsion of the section.
SAFIR can calculate this stiffness.

PRINCIPLE

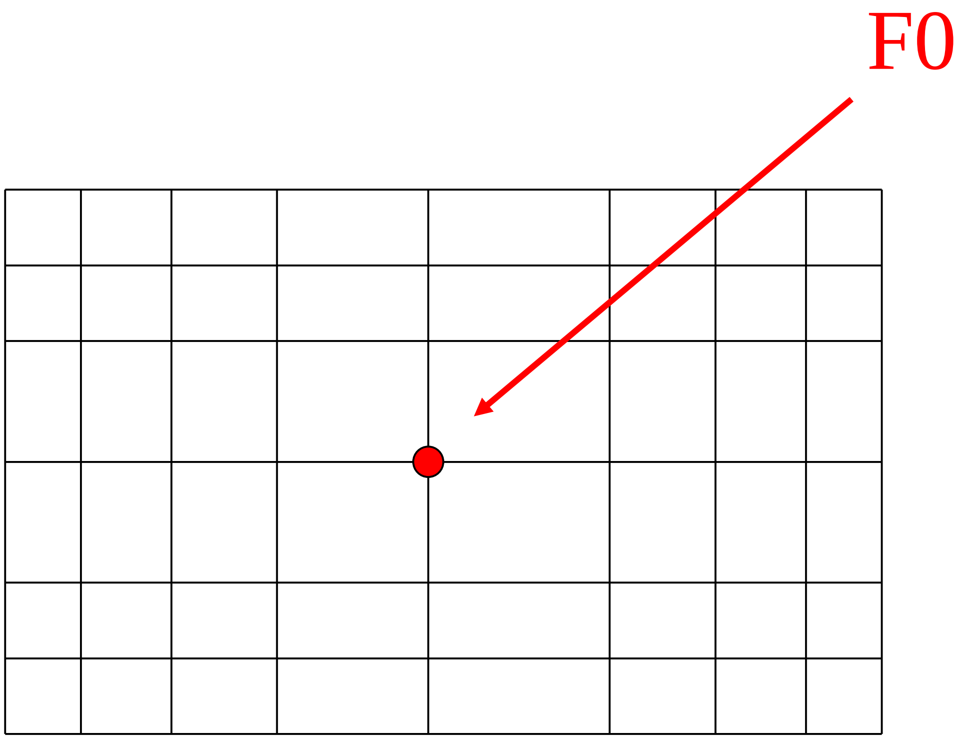
Use the same discretization as for the thermal analysis.
No fire curves.
Introduce appropriate material properties and fixations.
Introduce the result of the torsional analysis in the .TEM file.

Note: the calculated stiffness GJ is elastic, at room temperature. It is kept constant during the simulation.
$\Rightarrow$ possibility to reduce GJ arbitrarily.
$\Rightarrow$ The beam FE is not appropriate if torsion is the main load path.

Basic principle:SAFIR calculates the value of the warping function ω at the nodes



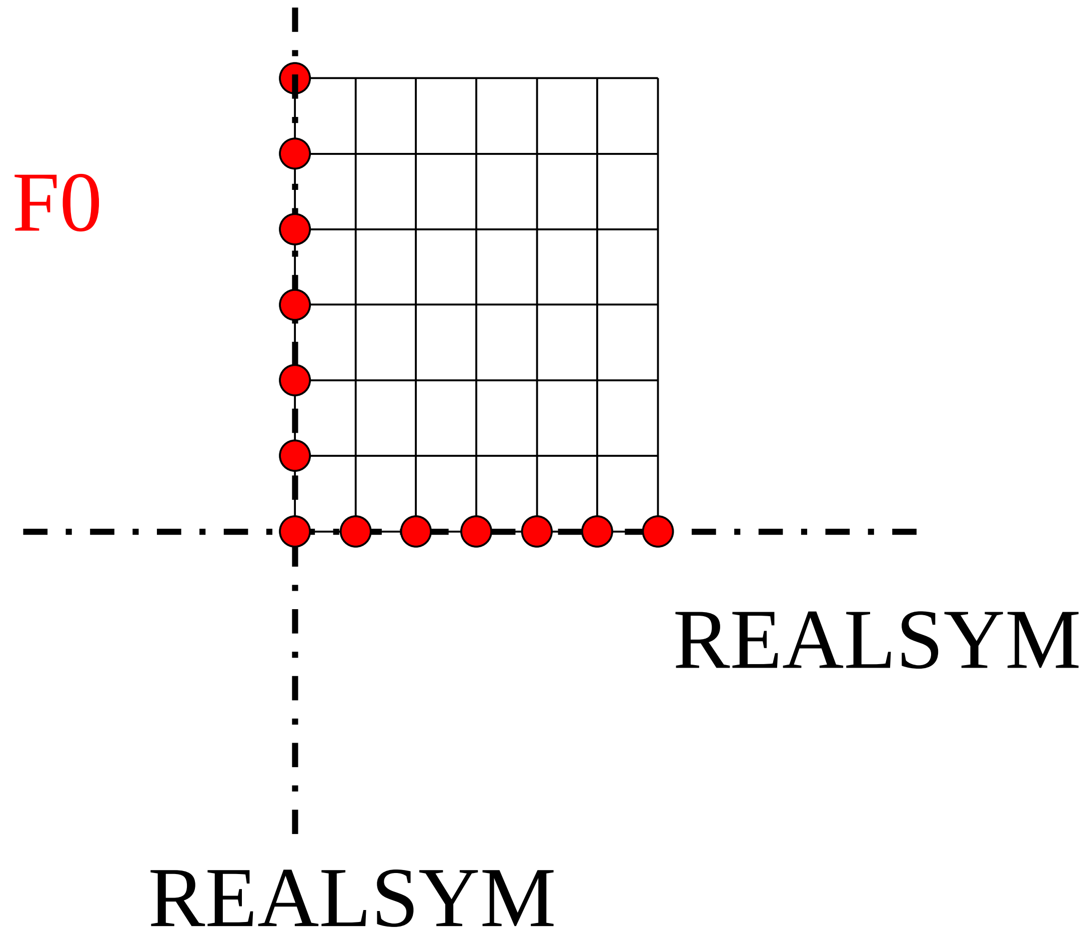
The derivatives of the warping function dω/dy and dω/dz are used to compute the stiffness in torsion GJ.
=> The solution can vary by a constant and GJ remains the same.



But, in order to have to correct behaviour, it is NECESSARY to fix the solution to 0 on the center of rotation.

Note: a check has been added and SAFIR will not run if there is no single node fixed (check the .OUT file of the torsion analysis if you performed the torsion analysis at the same time as the thermal analysis in GiD)

If you use symmetries, the warping function must be set to 0 on the axes of symmetry.



1) Local system of coordinates

2) Degrees of freedom

3) The basic equations

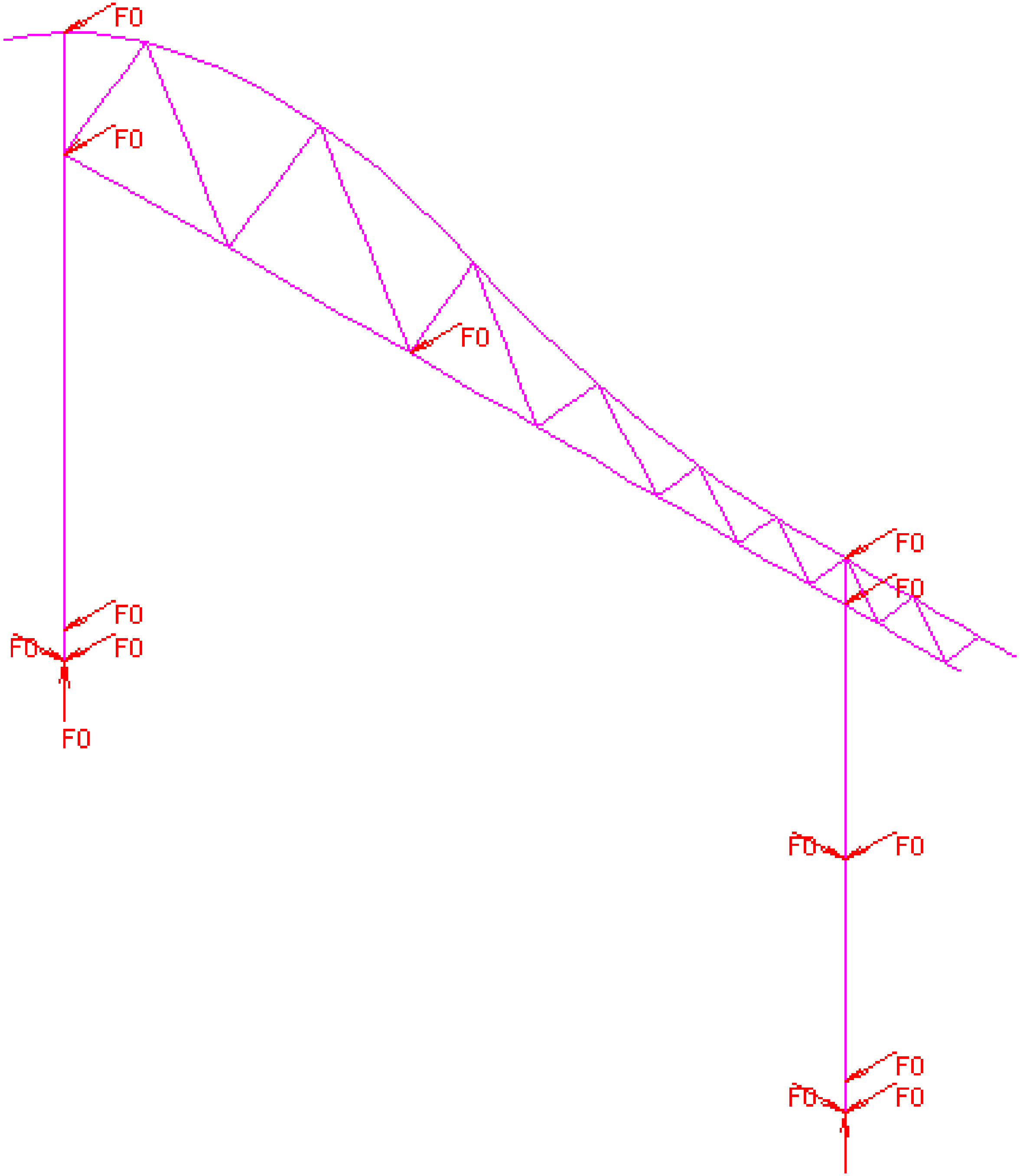
4) Material properties

5) Numerical integration on the length

6) Numerical integration on the surface

7) Stiffness in torsion

8) Examples



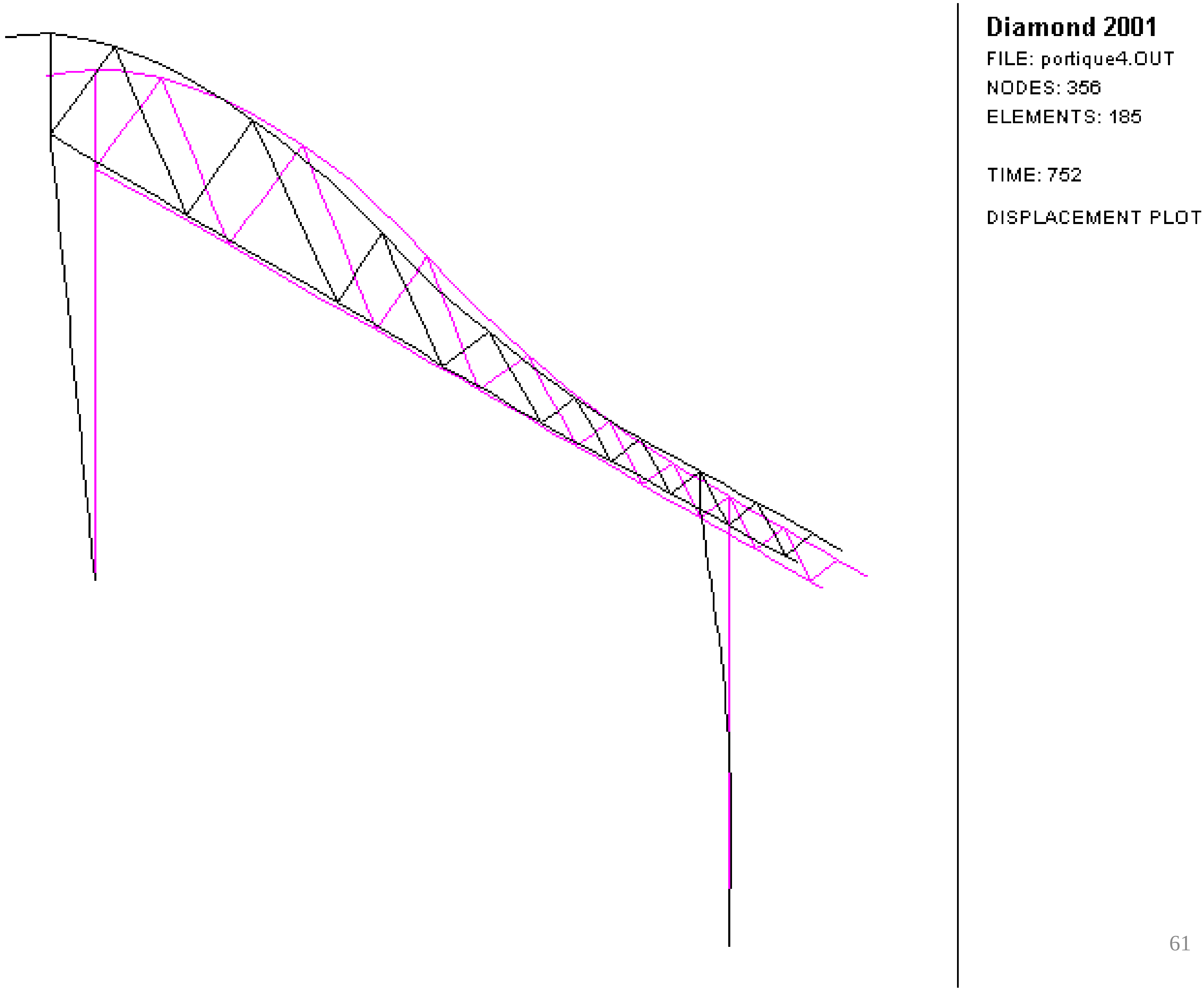
Diamond 2001

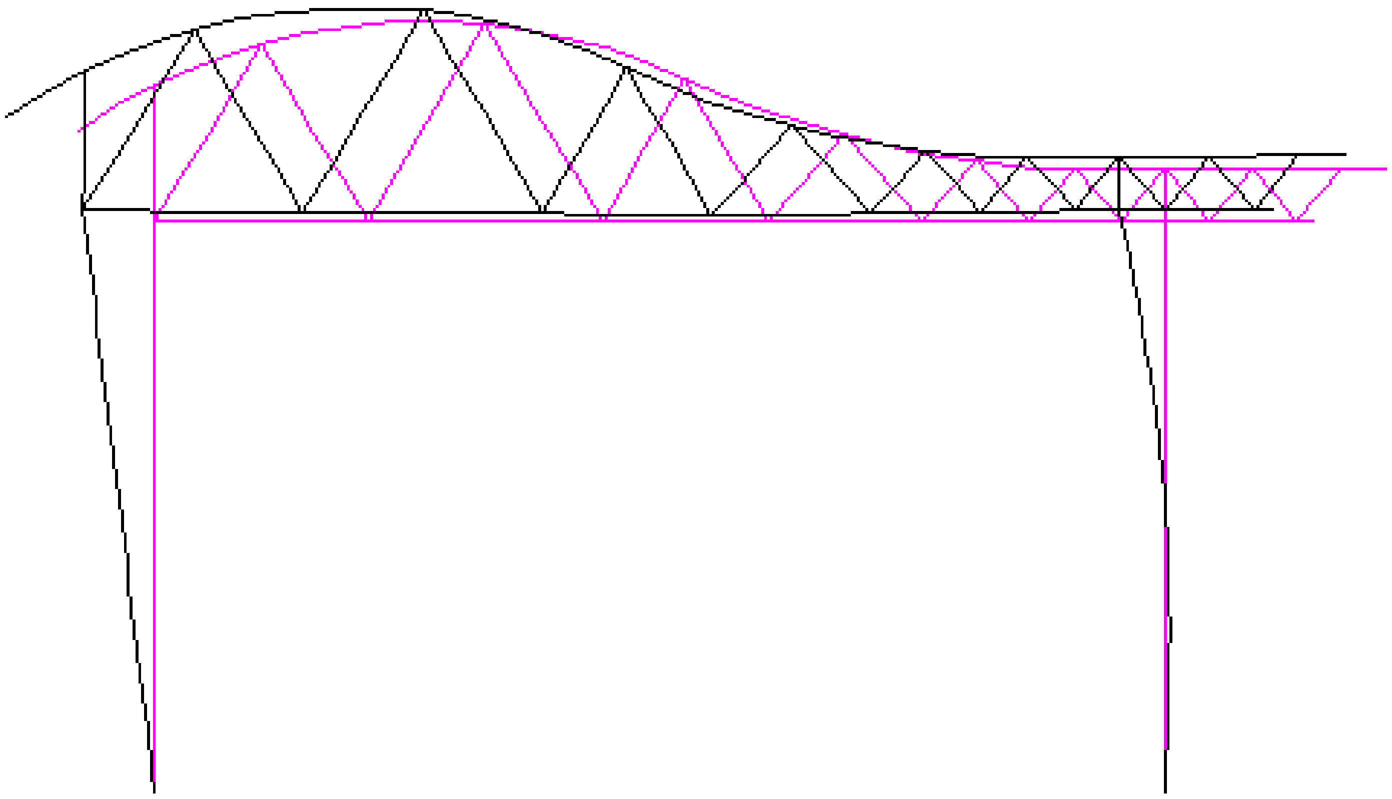
FILE: portique4.OUT

NODES: 356

ELEMENTS: 185

IMPOSED DOF PLOT


Diamond 2001
FILE: portique4.OUT
NODES: 356
ELEMENTS: 185
TIME: 752
DISPLACEMENT PLOT



Diamond 2001

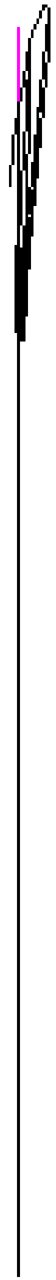
FILE: portique4.OUT

NODES: 356

ELEMENTS: 185

TIME: 752

DISPLACEMENT PLOT



Diamond 2001

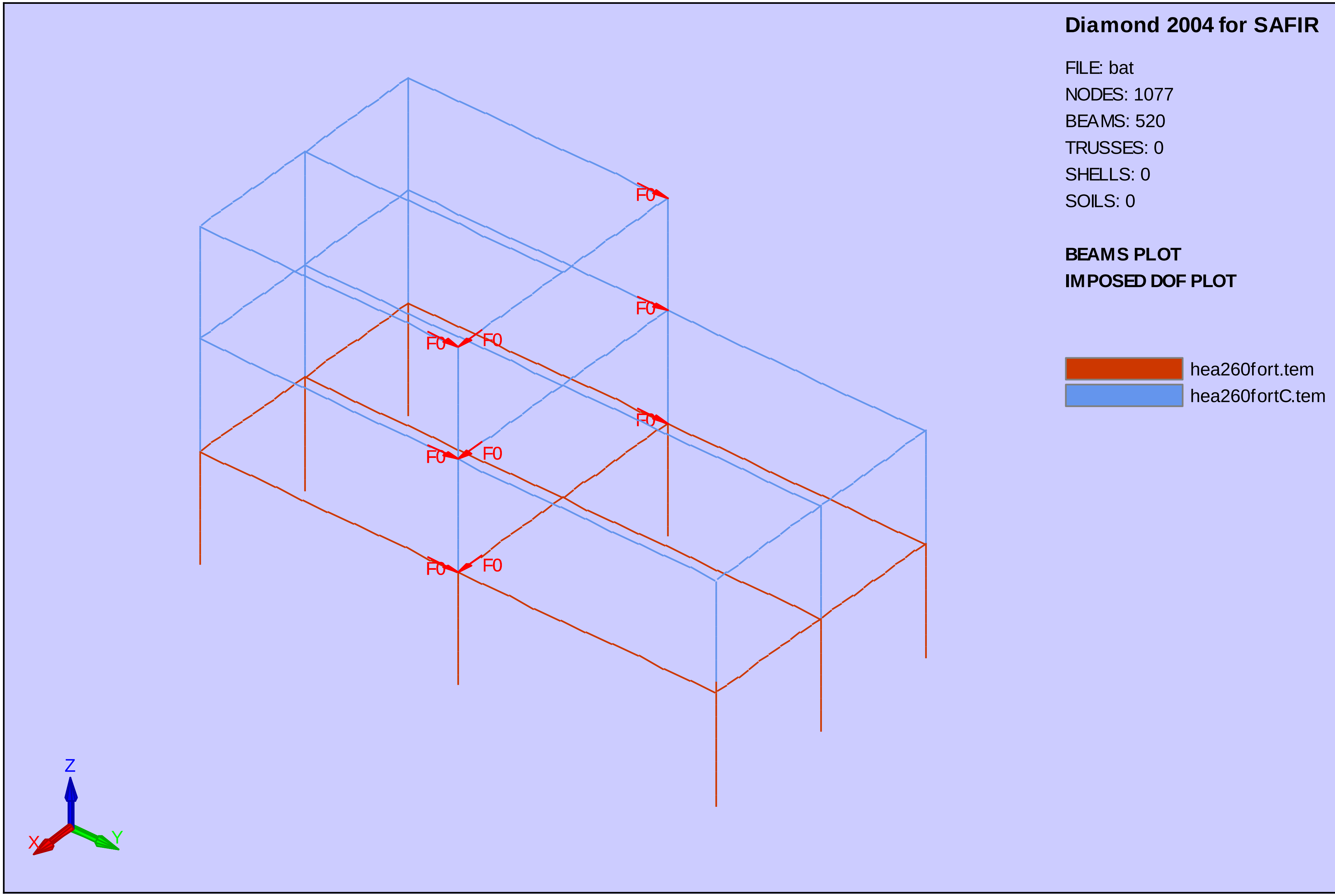
FILE: portique4.OUT

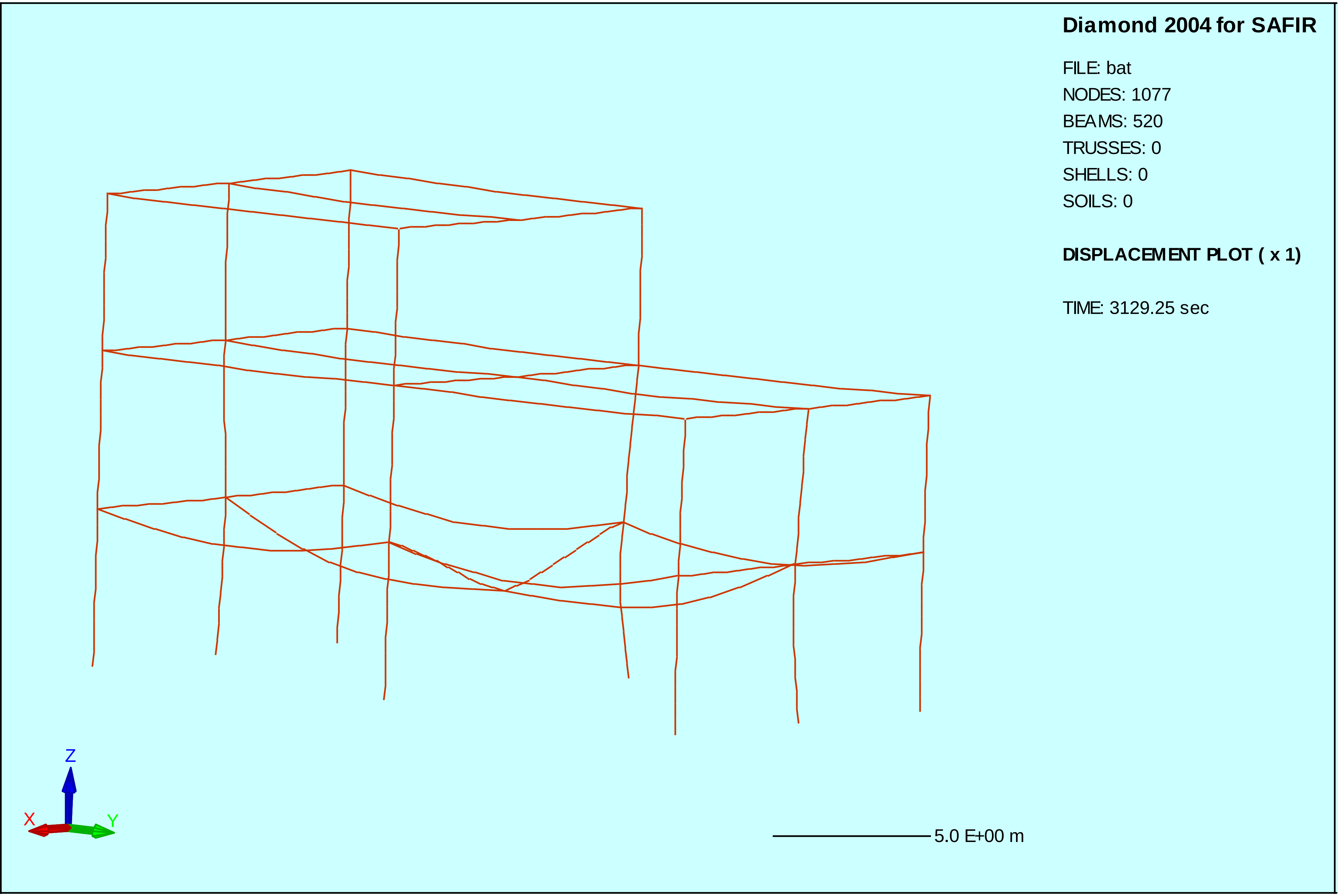
NODES: 356

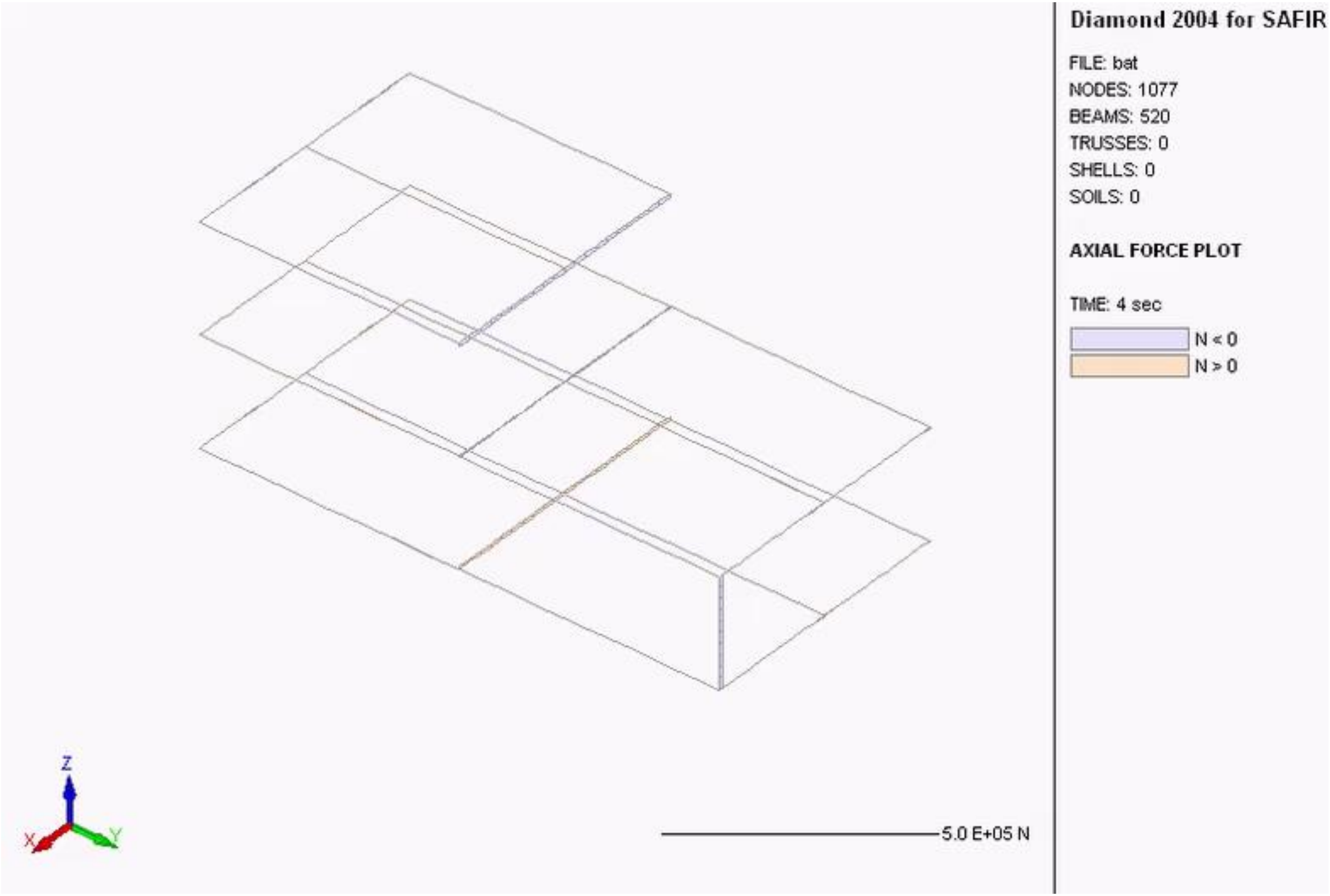
ELEMENTS: 185

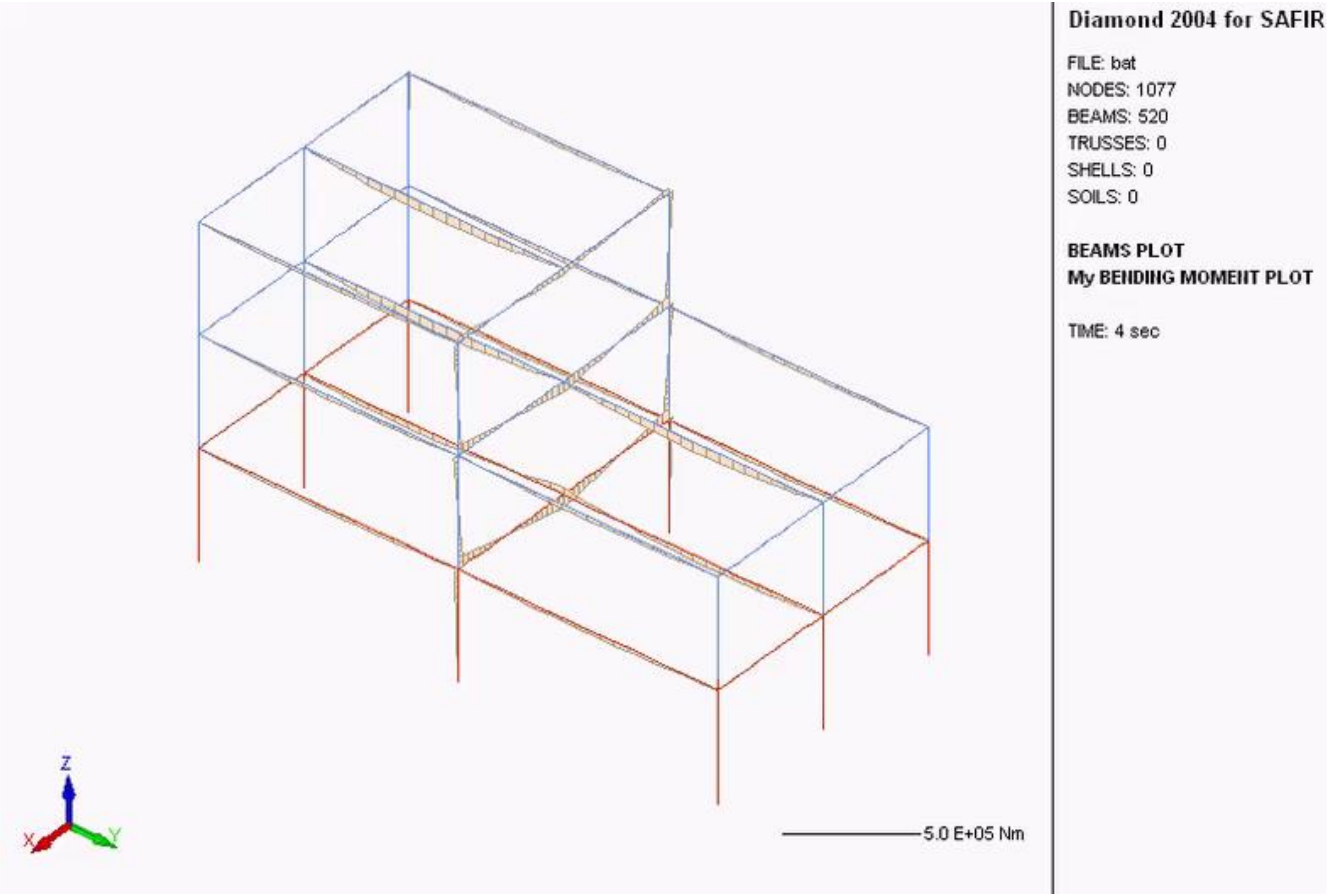
TIME: 752

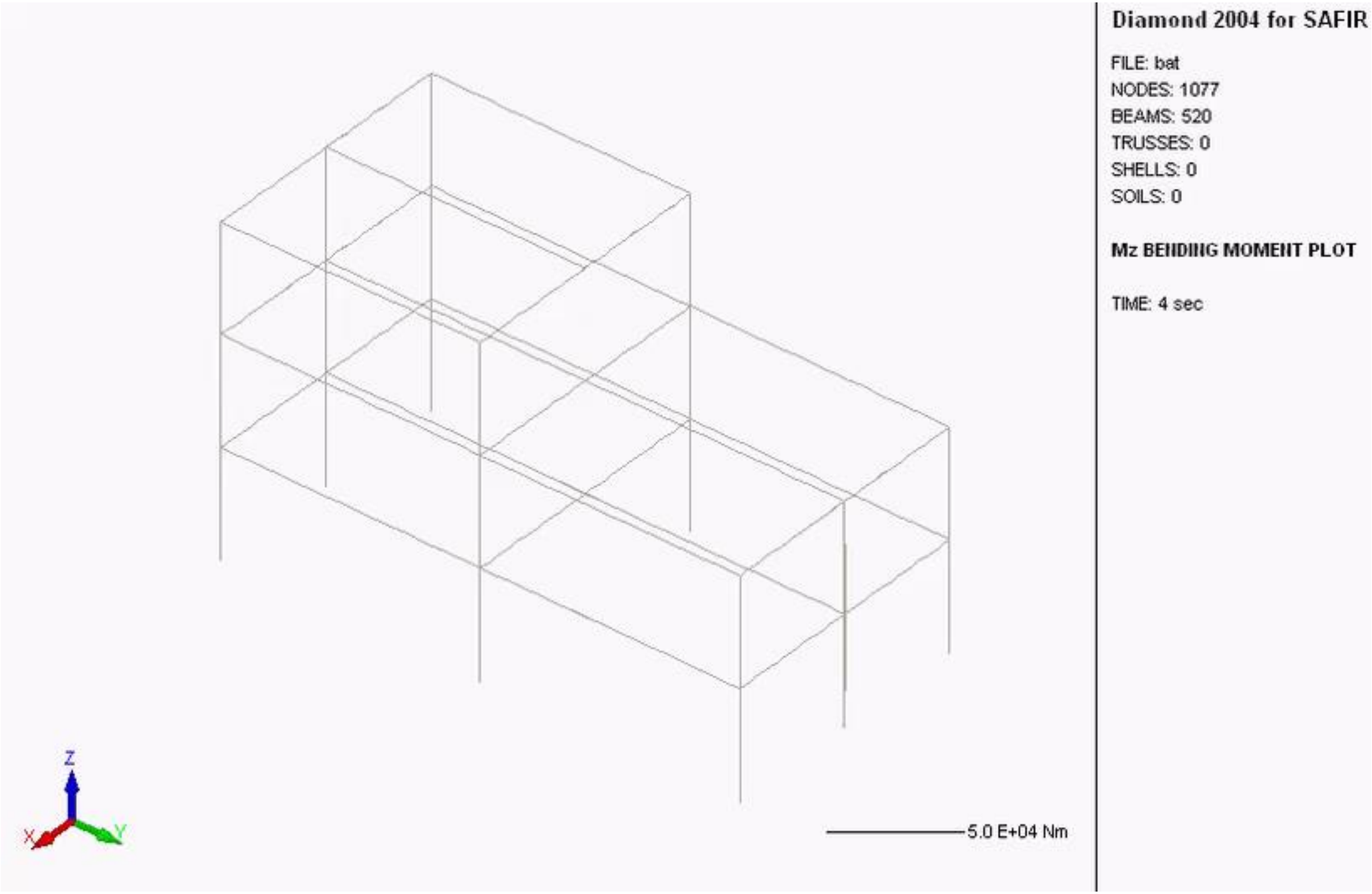
DISPLACEMENT PLOT

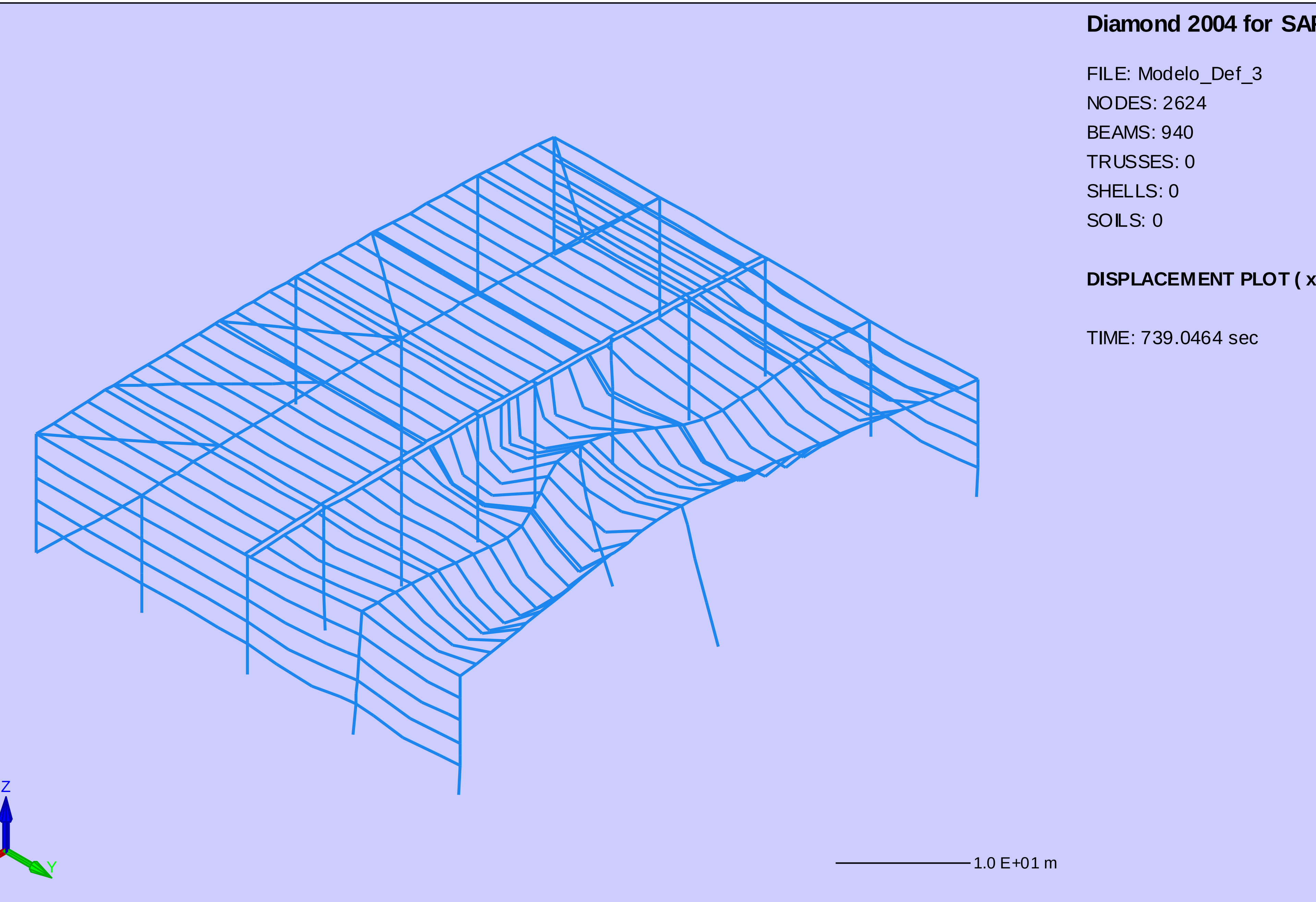

Diamond 2004 for SAFIR
FILE: bat
NODES: 1077
BEAMS: 520
TRUSSES: 0
SHELLS: 0
SOILS: 0
BEAMS PLOT
IMPOSED DOF PLOT
hea260fort.tem
hea260fortC.tem
F0
Z
X
Y


Diamond 2004 for SAFIR
FILE: bat
NODES: 1077
BEAMS: 520
TRUSSES: 0
SHELLS: 0
SOILS: 0
DISPLACEMENT PLOT (x 1)
TIME: 3129.25 sec
Z
X
Y
5.0 E+00 m


Diamond 2004 for SAFIR
FILE: bat
NODES: 1077
BEAMS: 520
TRUSSES: 0
SHELLS: 0
SOILS: 0
AXIAL FORCE PLOT
TIME: 4 sec
N < 0
N > 0
5.0 E+05 N
Z
X
Y

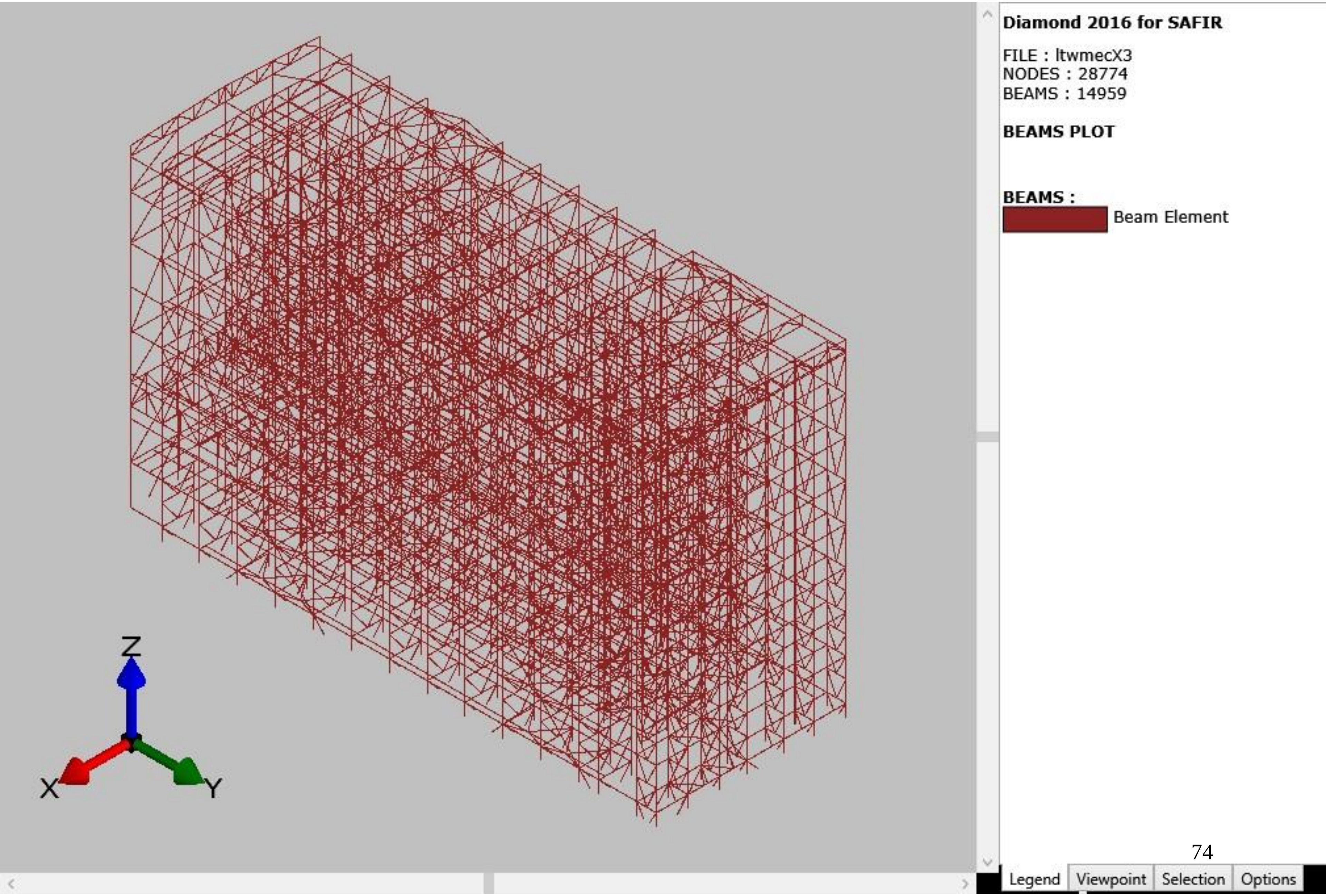

Diamond 2004 for SAFIR
FILE: bat
NODES: 1077
BEAMS: 520
TRUSSES: 0
SHELLS: 0
SOILS: 0
BEAMS PLOT
My BENDING MOMENT PLOT
TIME: 4 sec
5.0 E+05 Nm
Z
X
Y


Diamond 2004 for SAFIR
FILE: bat
NODES: 1077
BEAMS: 520
TRUSSES: 0
SHELLS: 0
SOILS: 0
Mz BENDING MOMENT PLOT
TIME: 4 sec
5.0 E+04 Nm
Z
X
Y


Diamond 2004 for SA
FILE: Modelo_Def_3
NODES: 2624
BEAMS: 940
TRUSSES: 0
SHELLS: 0
SOILS: 0
DISPLACEMENT PLOT (
TIME: 739.0464 sec
1.0 E+01 m
Z
Y

Project FLUMILOG (CTICM)

Steel rack structure (courtesy Angel Guerrero Castells, IGNIA, Spain)



Thank you

For any further information please contact:
safir@uliege.be

Jean Marc Franssen & Thomas Gernay